Communicating Finite-State Machines, First-Order Logic, and Star-Free Propositional Dynamic Logic

Benedikt Bollig, Marie Fortin, Paul Gastin

CNRS, LSV, ENS Paris-Saclay

CAALM
Chennai Mathematical Institute
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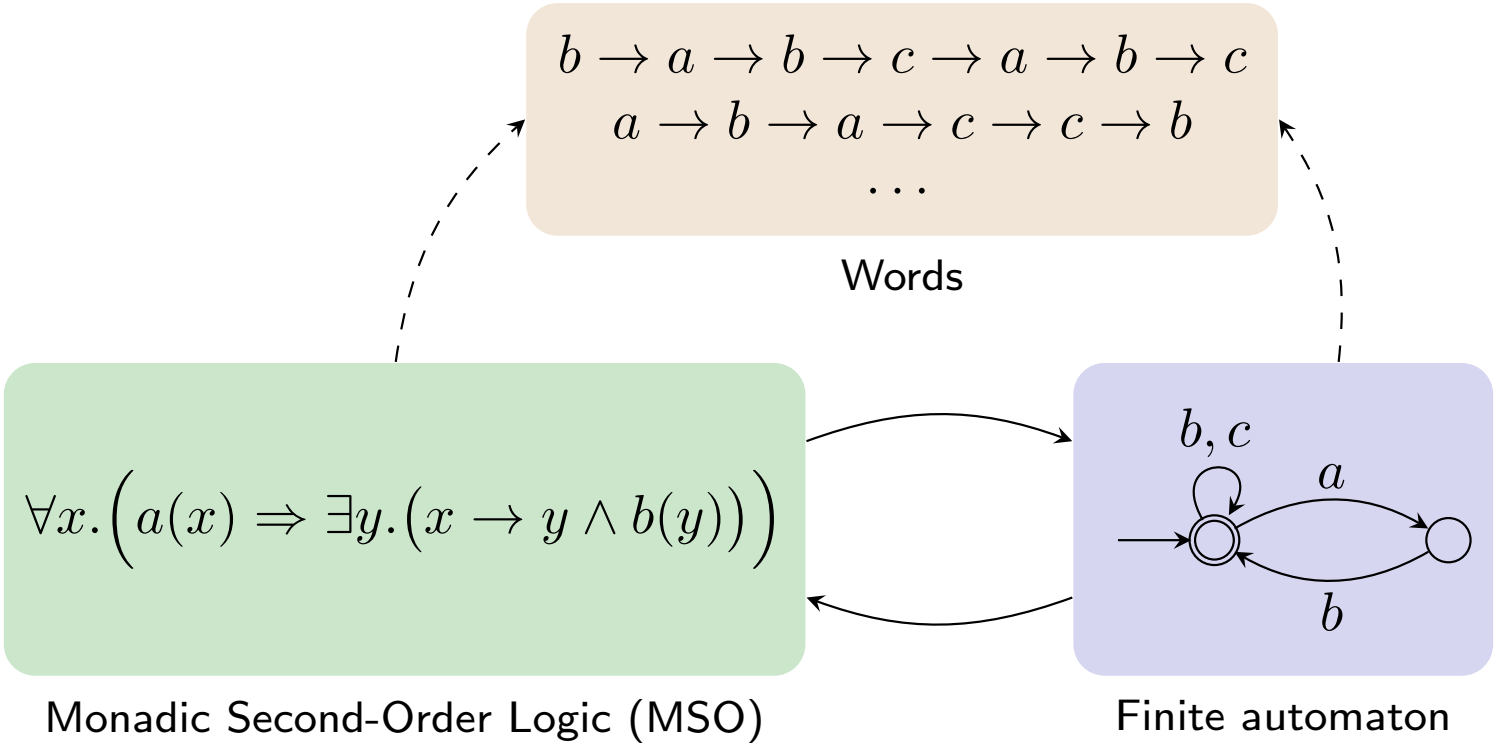
Slides courtesy of Marie Fortin.

Büchi-Elgot-Trakhtenbrot Theorem ('60s)



$\mathsf{MSO}[\to] = \text{Finite Automata}$

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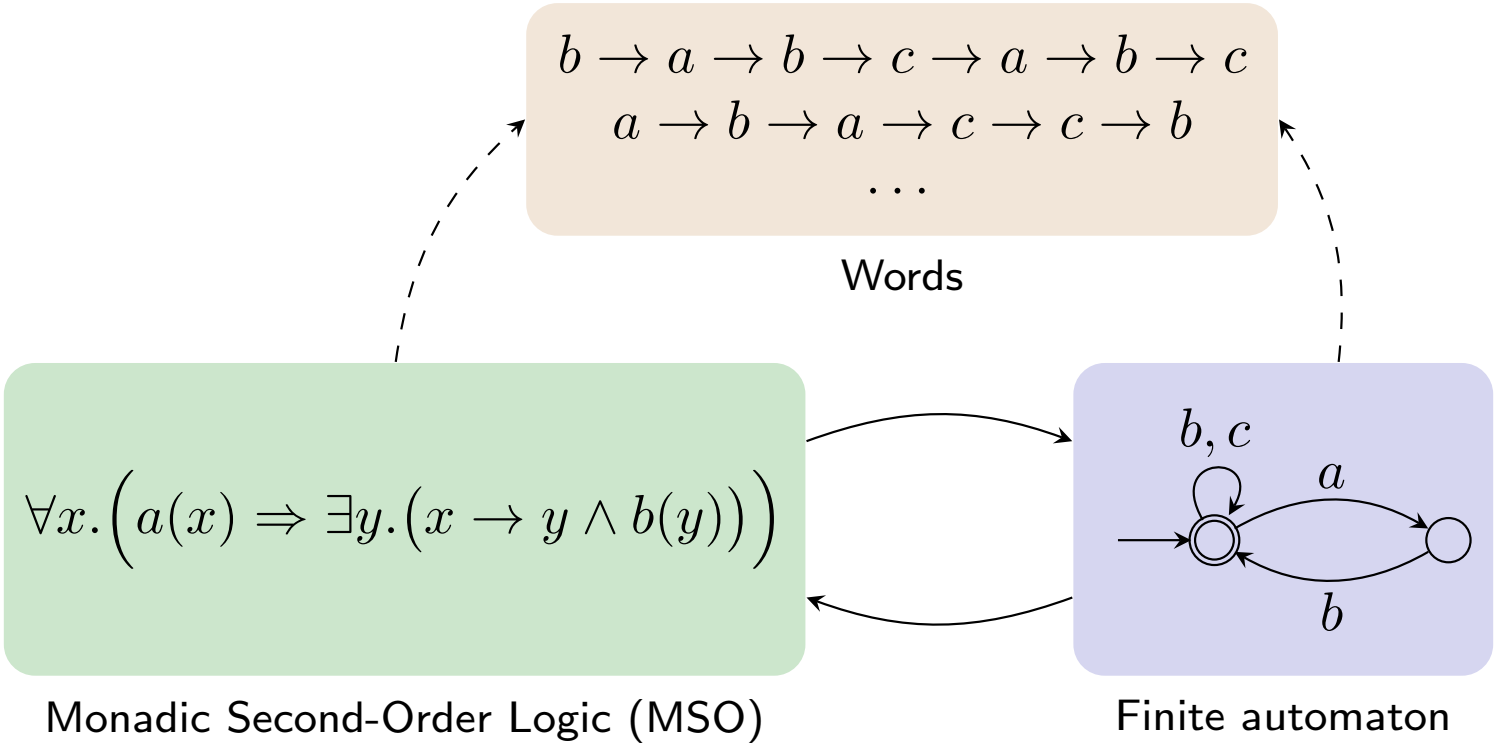

$$\mathsf{MSO}[\to] = \text{Finite Automata} = \mathsf{EMSO}[\to]$$

$\exists X_0 \dots \exists X_n.\varphi$ with $\varphi \in \mathsf{FO}[\to]$

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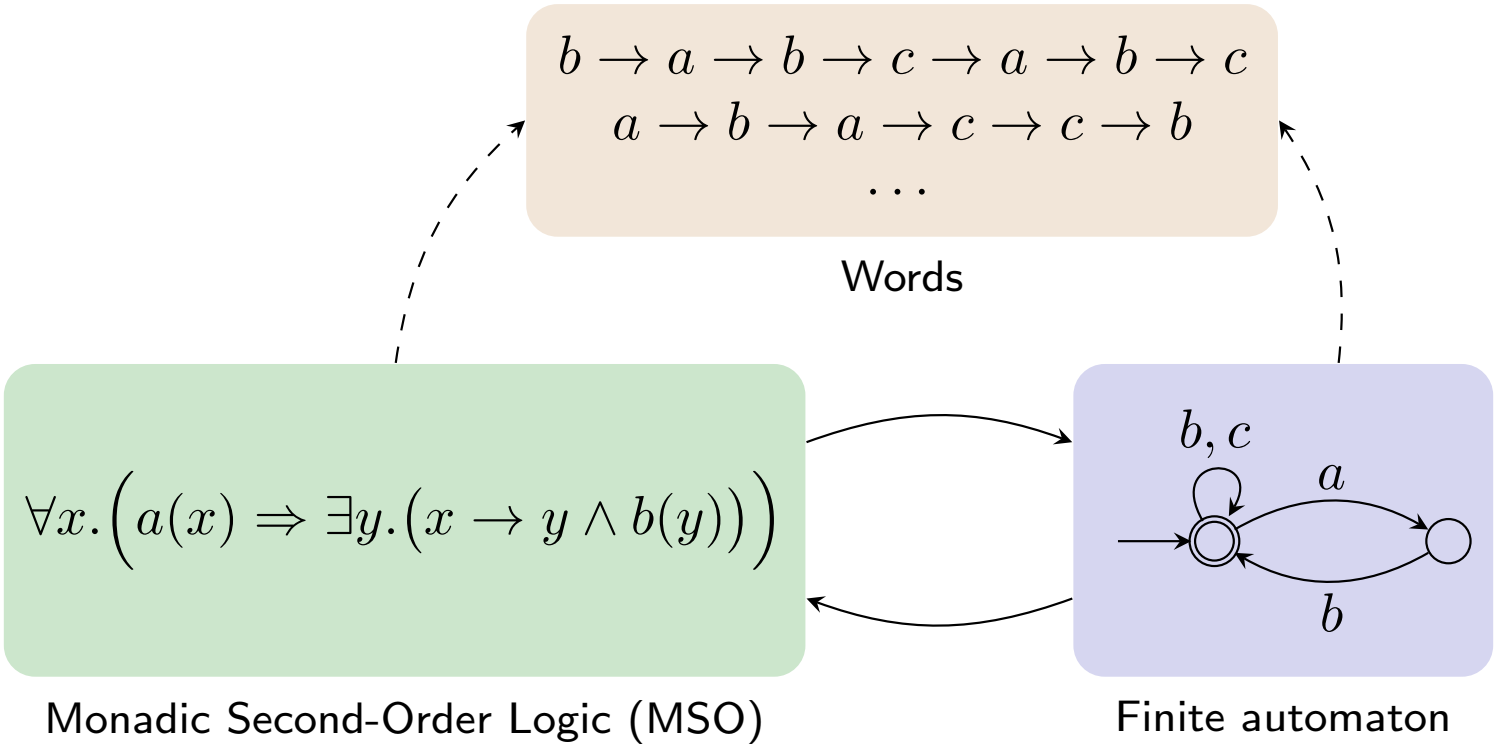

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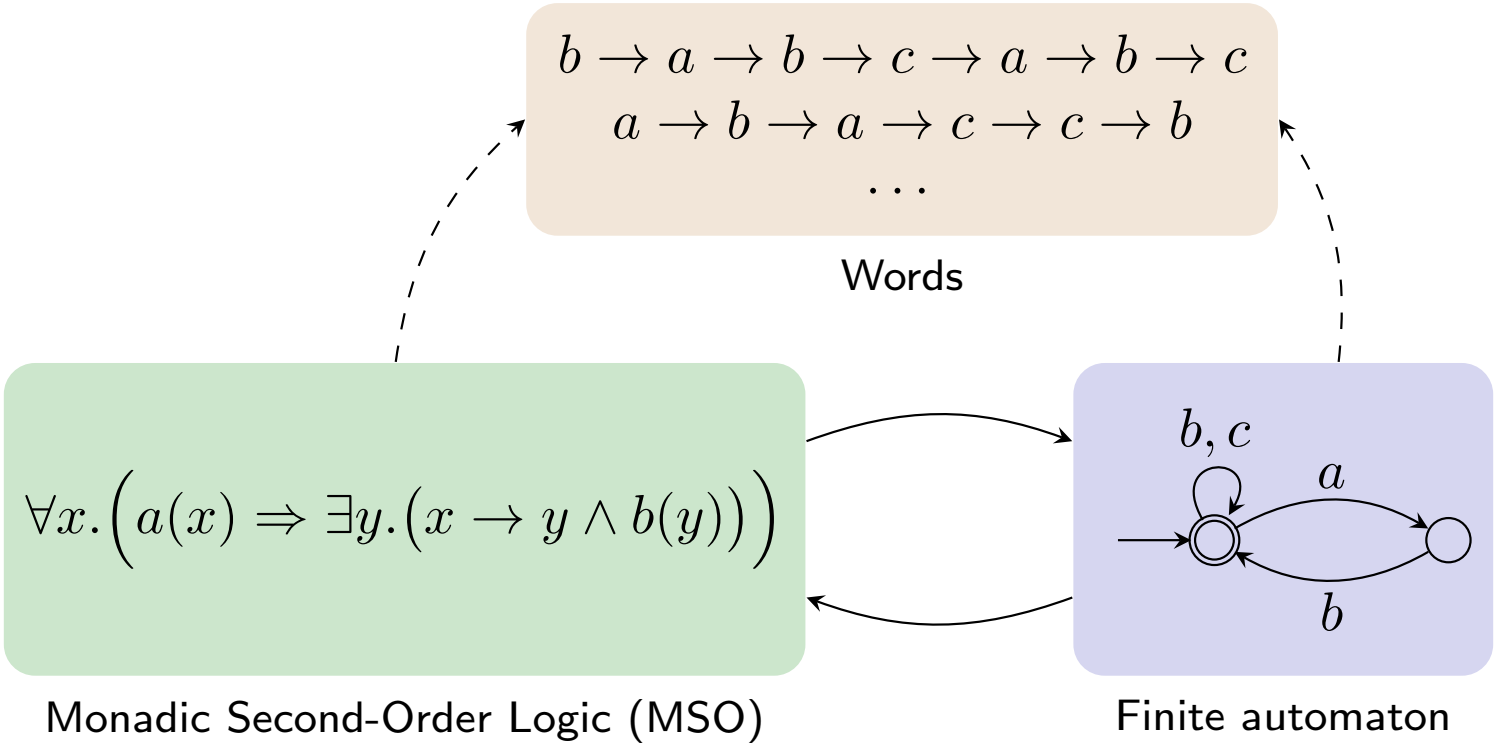
Extended to trees [Thatcher-Wright '68], Mazurkiewicz traces [Thomas '90], nested words [Alur-Madhusudan '04], data words [Bojańczyk et al. '06], weighted automata [Droste-Gastin '05], ...

Büchi-Elgot-Trakhtenbrot Theorem ('60s)



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Goal: A Büchi-like theorem for message-passing systems

Büchi-Elgot-Trakhtenbrot Theorem ('60s)

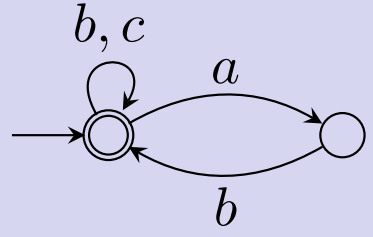
$$b \to a \to b \to c \to a \to b \to c$$
$$a \to b \to a \to c \to c \to b$$
$$\dots$$

Words

$\forall x.\Big(a(x) \Rightarrow \exists y.\big(x \to y \wedge b(y)\big)\Big)$

Monadic Second-Order Logic (MSO)

Synthesis



Finite automaton

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The model

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- Fix finite set of processes and finite alphabet (e.g., $P = \{p, q, r\}$ and $\Sigma = \{a, b, c\}$)

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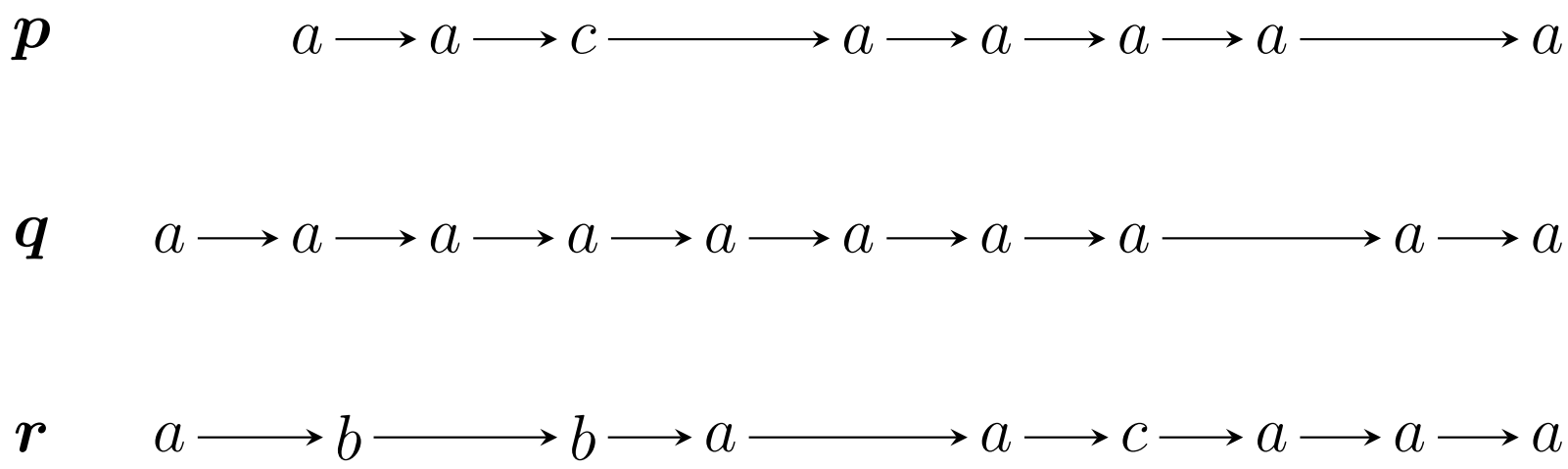

The model

- Fix finite set of processes and finite alphabet (e.g., $P = \{p, q, r\}$ and $\Sigma = \{a, b, c\}$)
- Reliable unbounded point-to-point FIFO channels

$\boldsymbol{p}$ $\quad a \longrightarrow a \longrightarrow c \longrightarrow\!\!\!\!\longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow\!\!\!\!\longrightarrow a$

$\boldsymbol{q}$ $\quad a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow a \longrightarrow\!\!\!\!\longrightarrow a \longrightarrow a$

$\boldsymbol{r}$ $\quad a \longrightarrow b \longrightarrow\!\!\!\!\longrightarrow b \longrightarrow a \longrightarrow\!\!\!\!\longrightarrow a \longrightarrow c \longrightarrow a \longrightarrow a \longrightarrow a$

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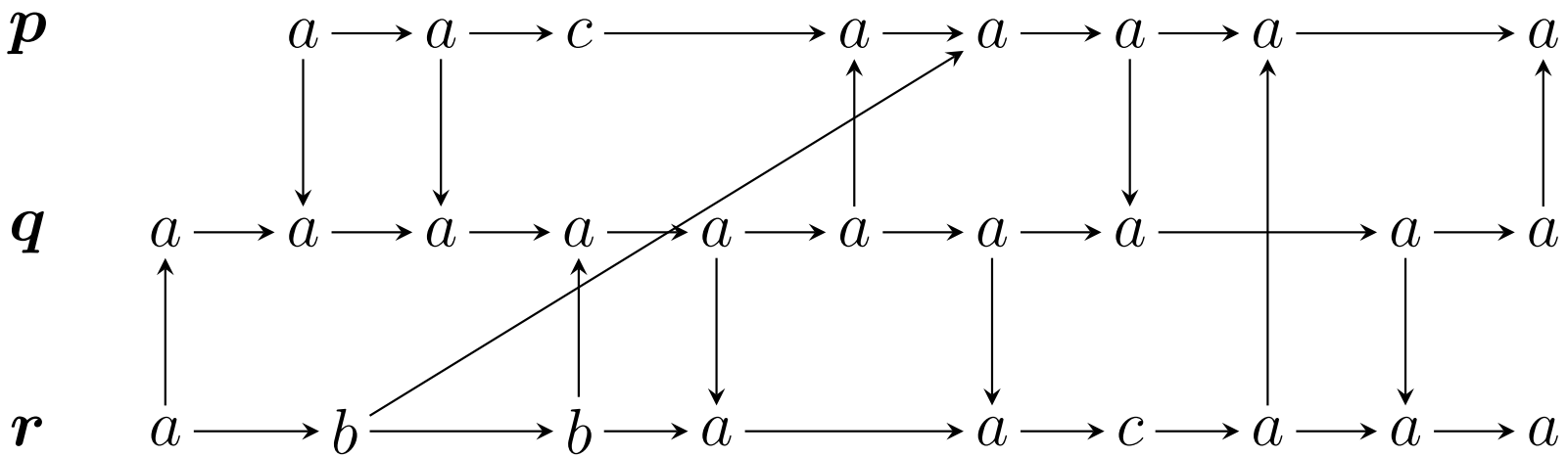

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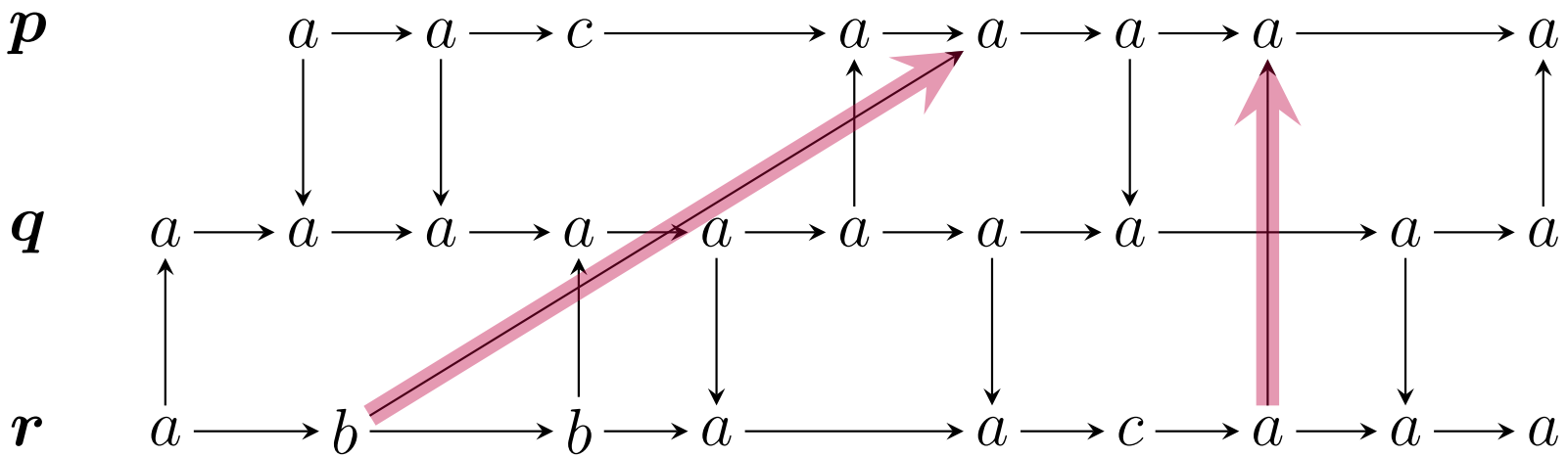

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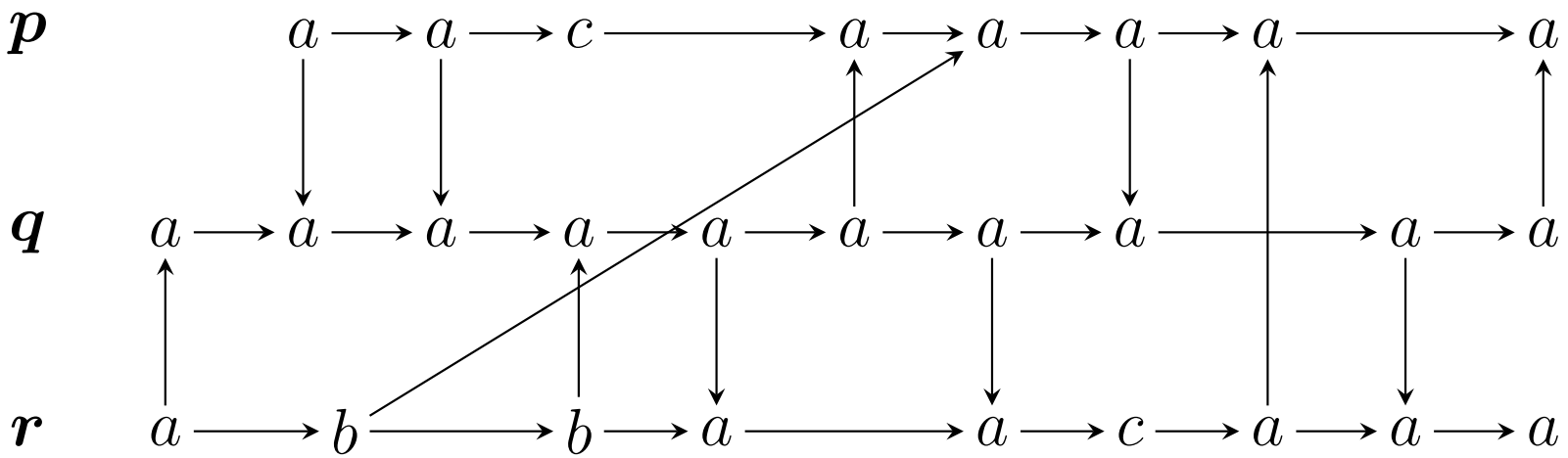
- Reliable unbounded point-to-point FIFO channels
- Partial order semantics: Message Sequence Charts (MSC)



Communicating finite-state machines (CFMs)

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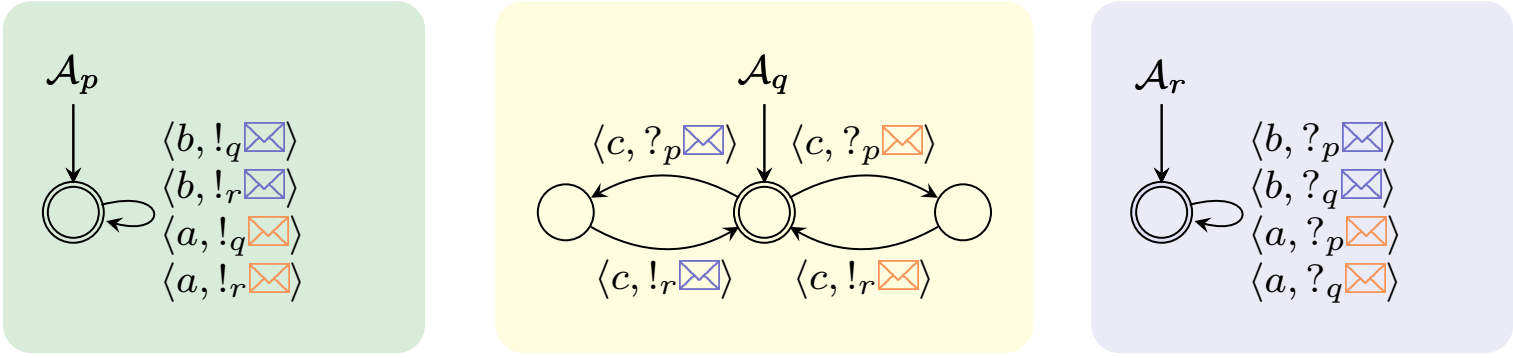

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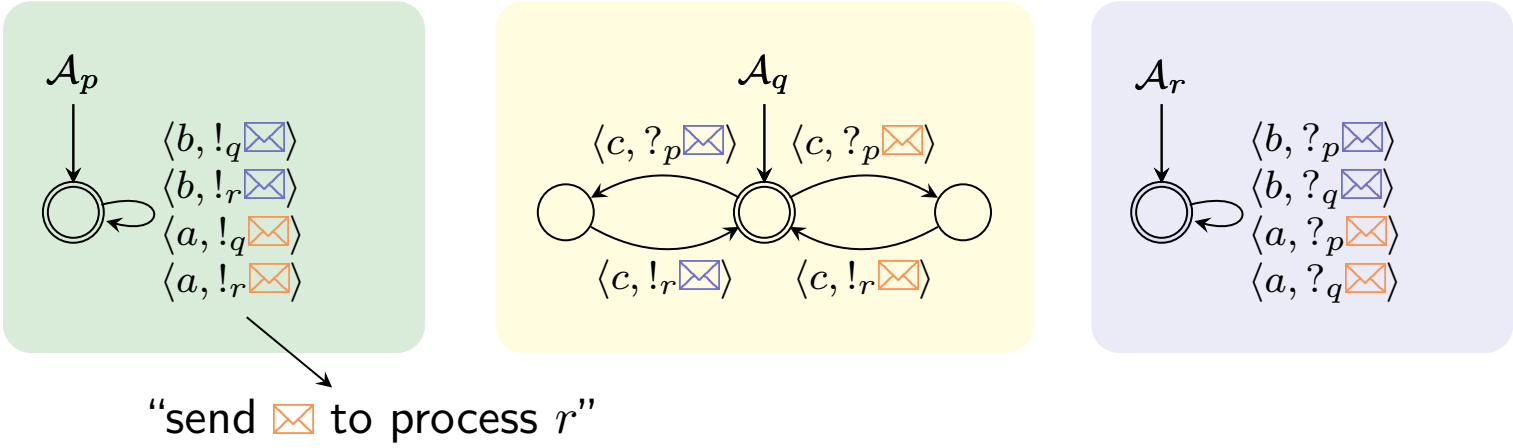

"send ✉ to process r"

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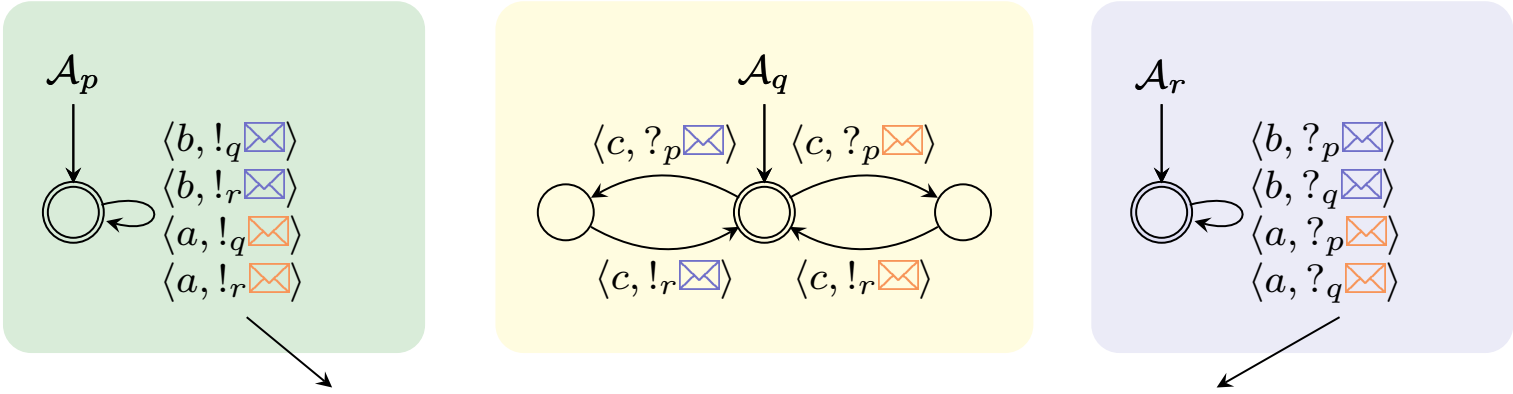

"send ✉ to process r" $\qquad$ "receive ✉ from process q"

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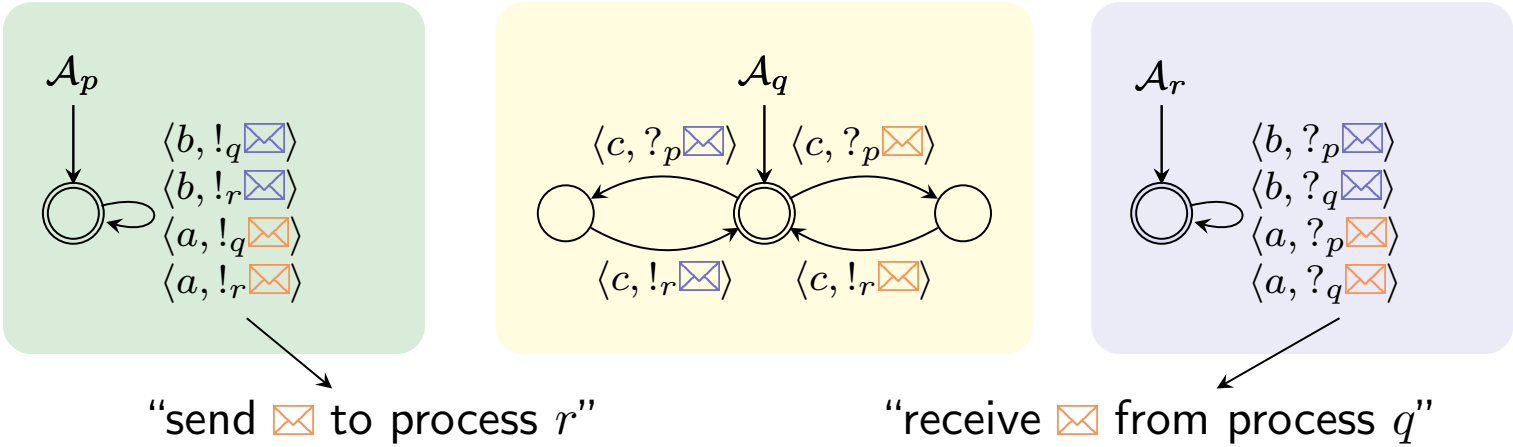
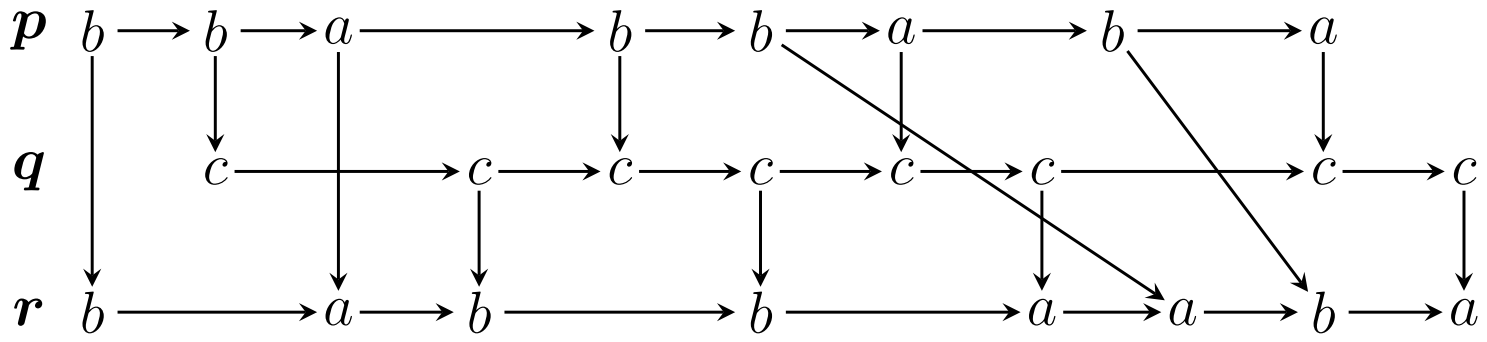


Run of CFMs on MSCs:



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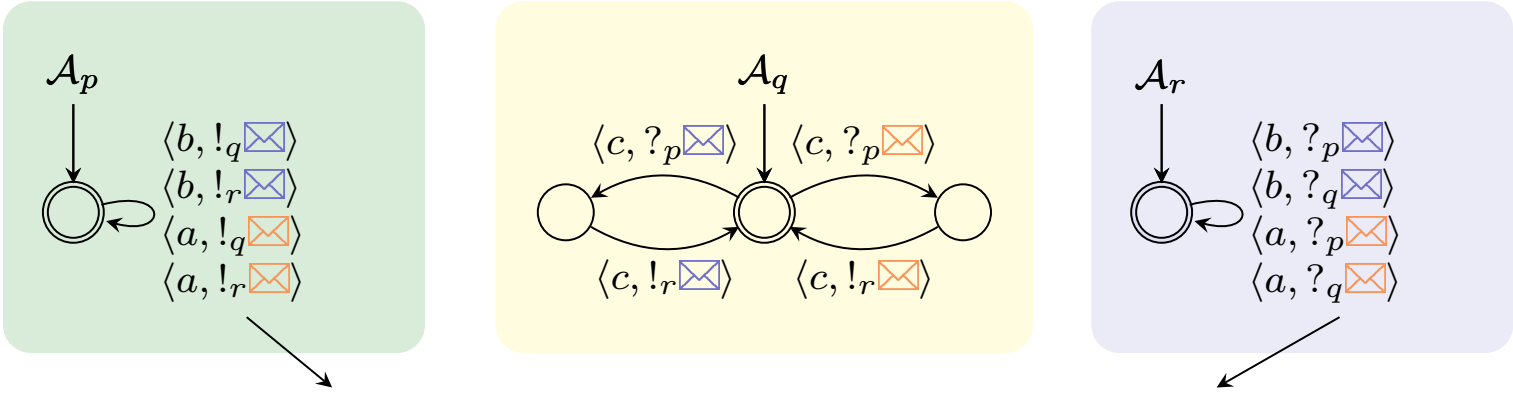

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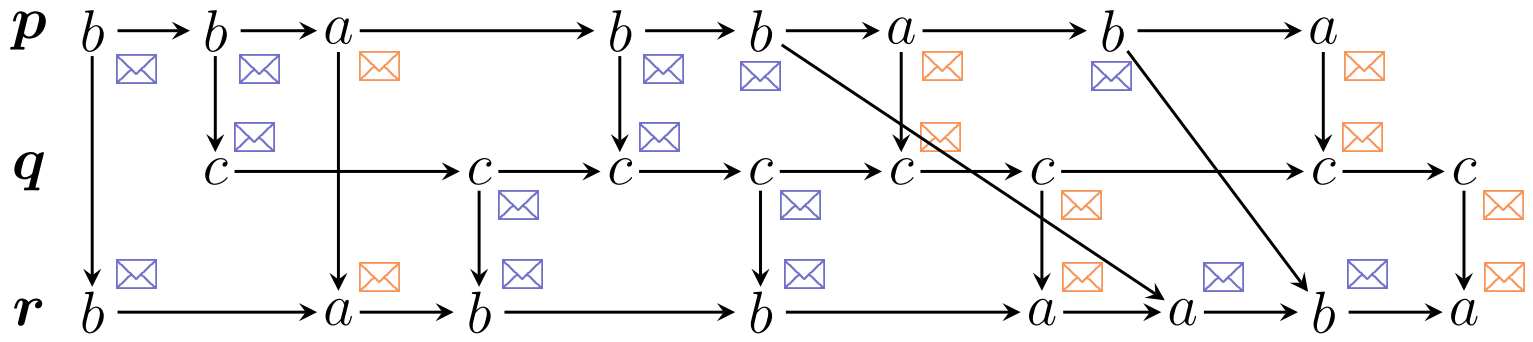

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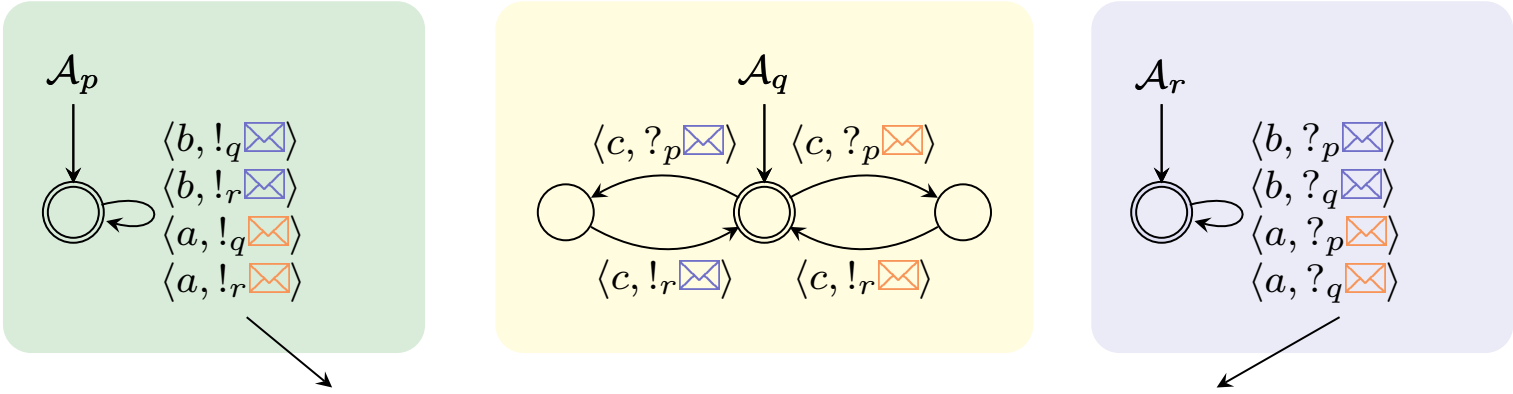

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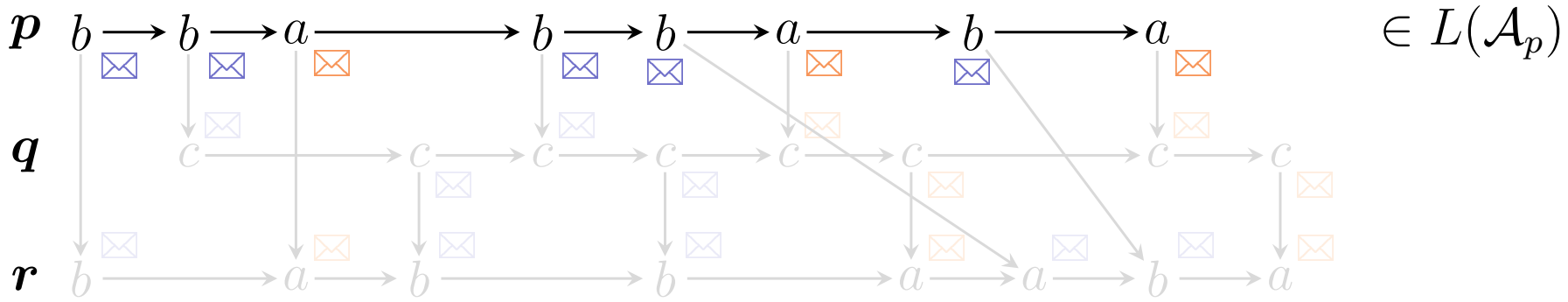

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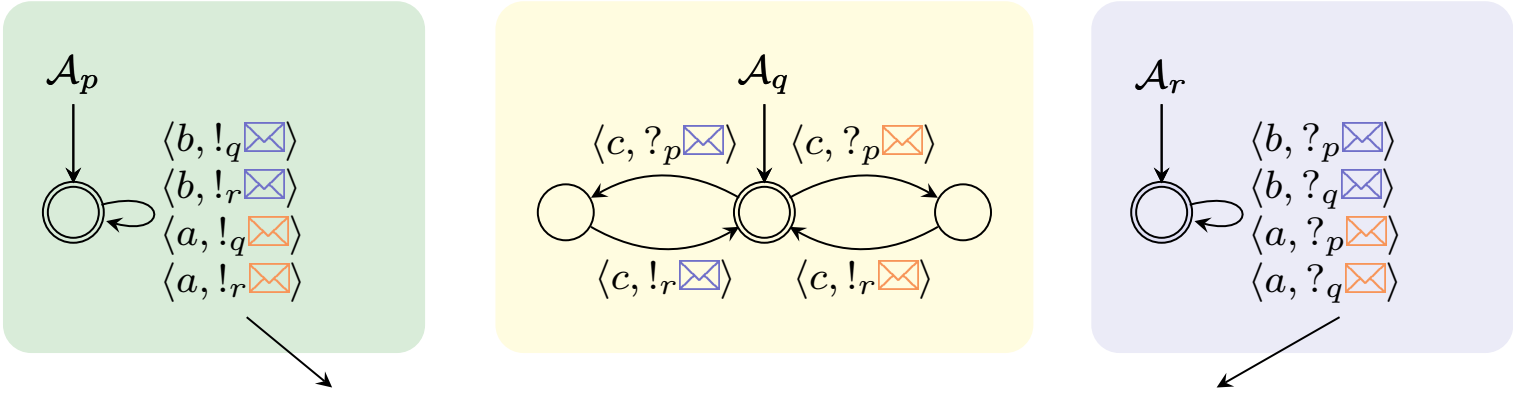

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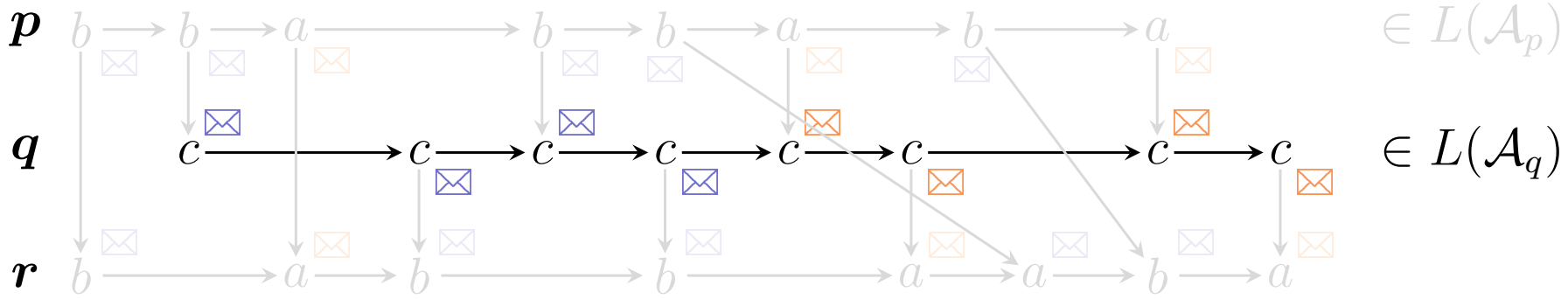

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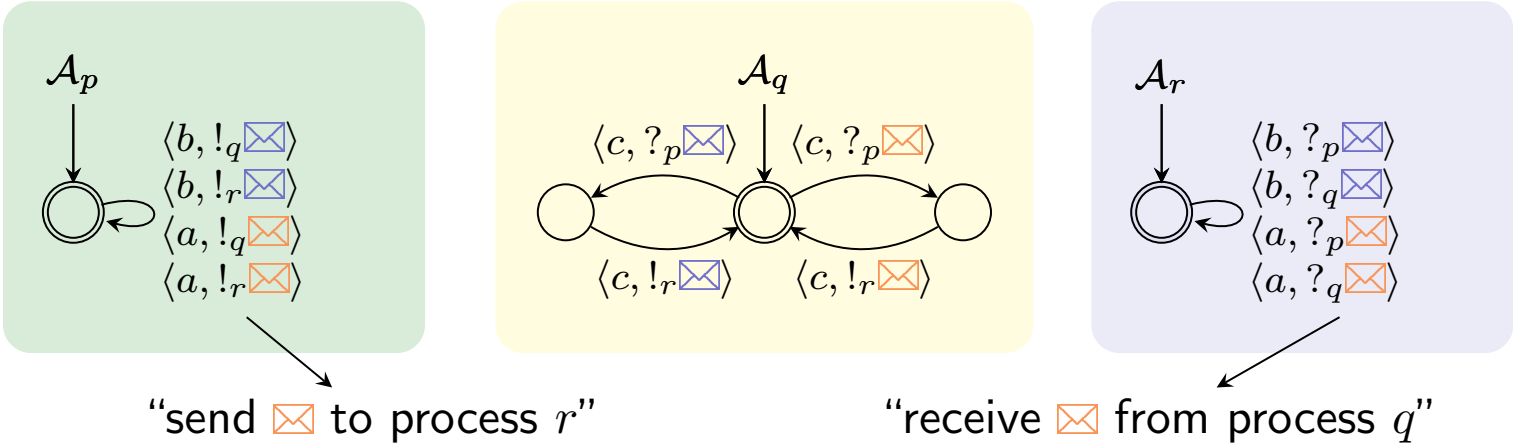

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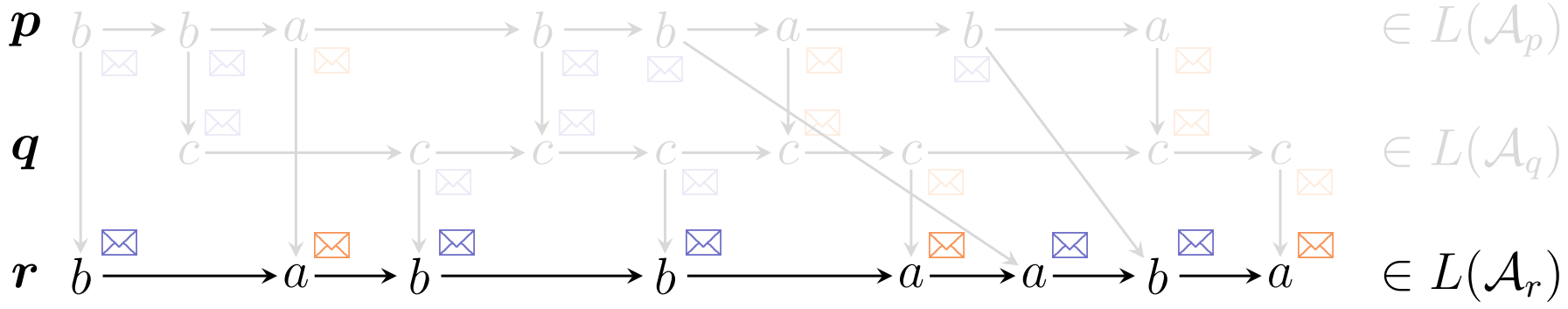

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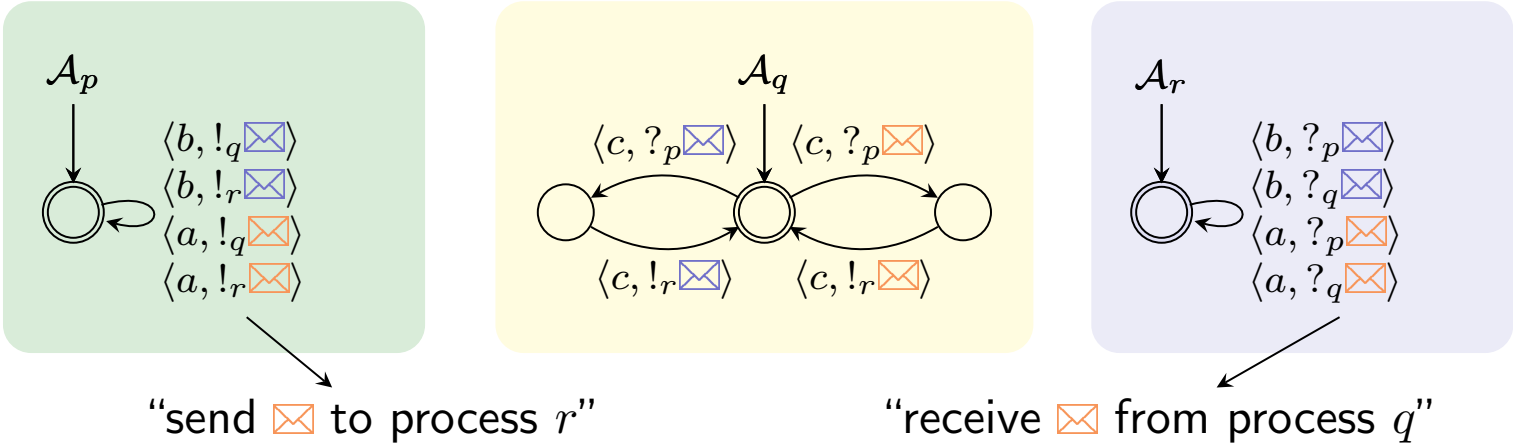

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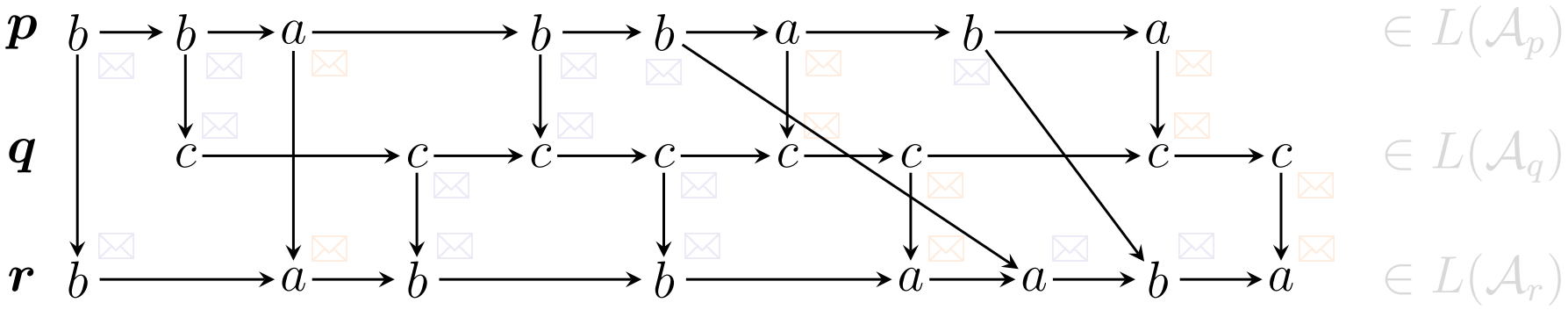

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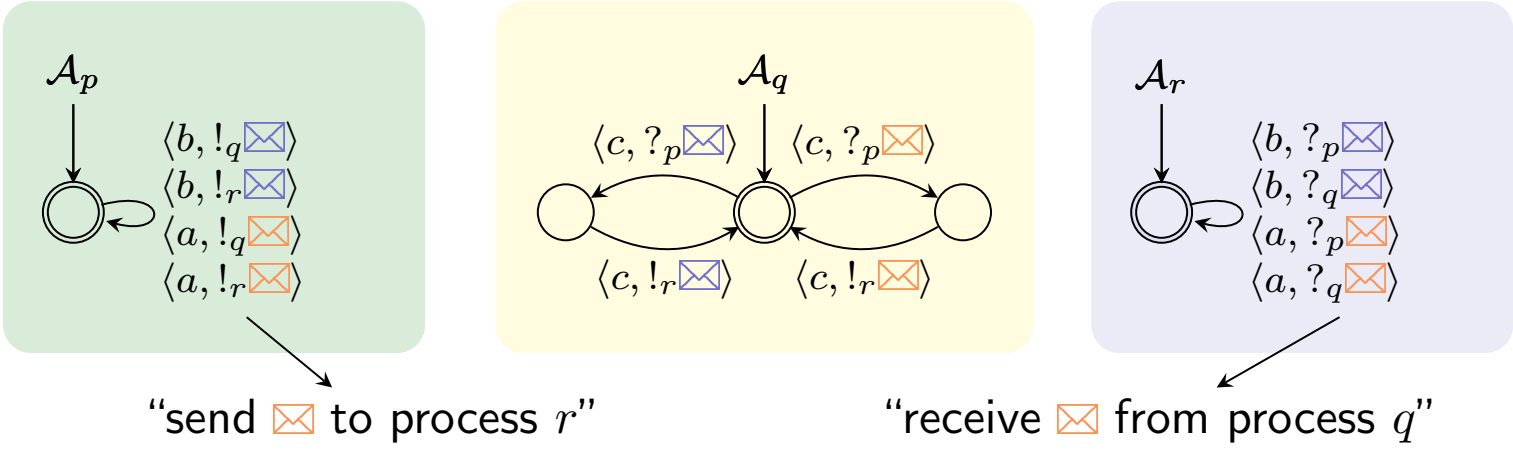


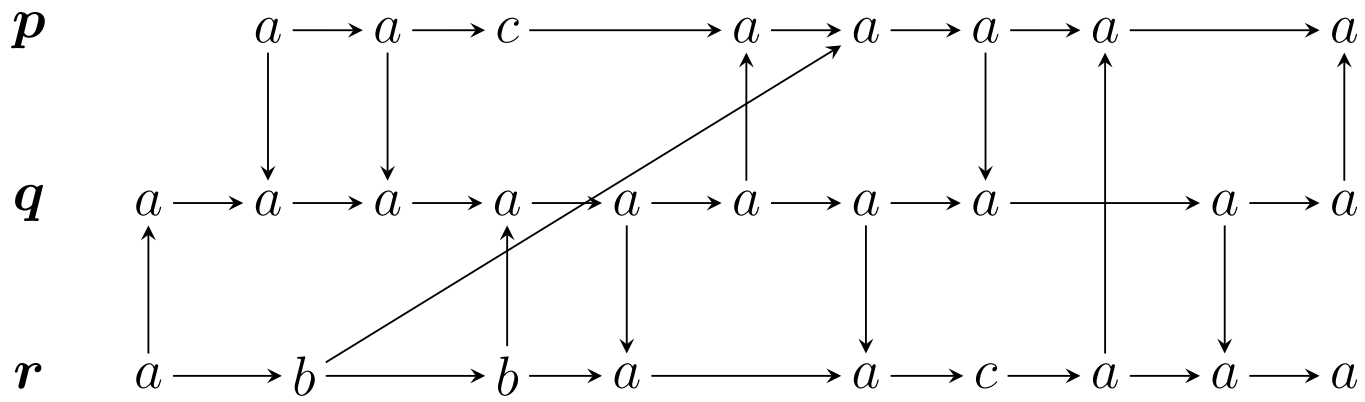
Remarks

- The emptiness problem for CFMs is **undecidable**.
- CFMs are inherently **non-deterministic**.

[Genest-Kuske-Muscholl '07]

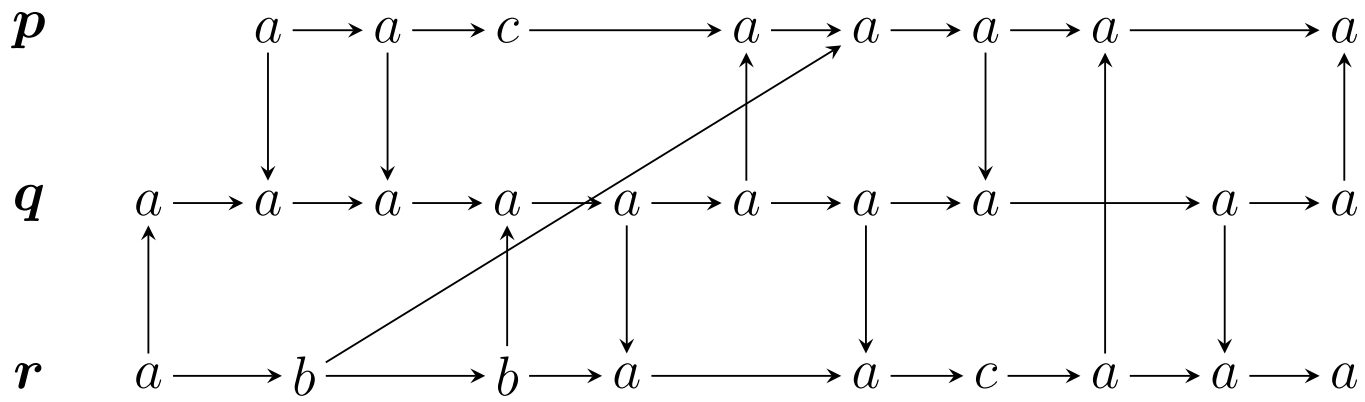
Monadic Second-Order Logic (MSO) over MSCs

$\varphi ::=$



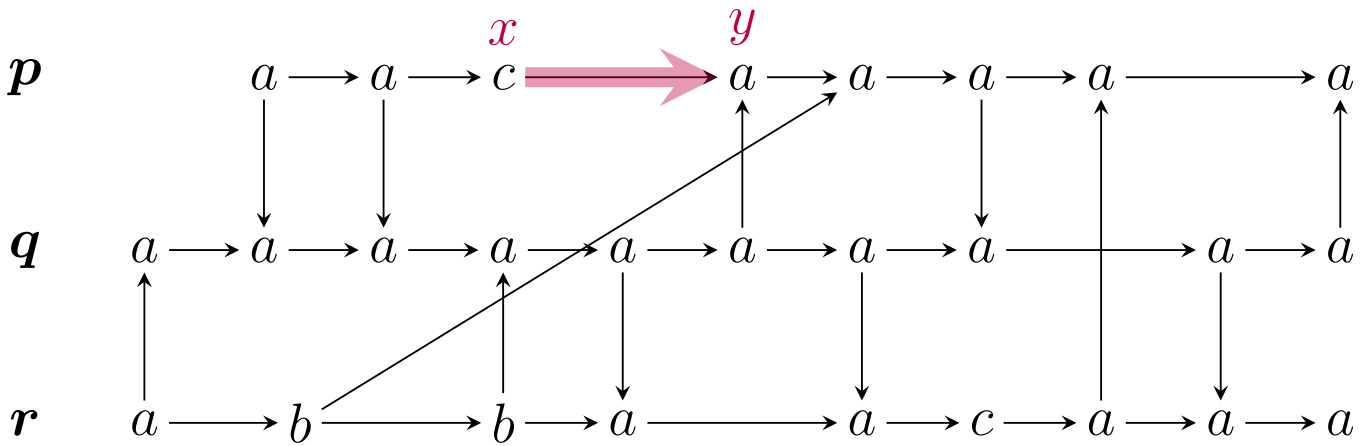
Monadic Second-Order Logic (MSO) over MSCs

$\varphi ::= \quad a(x) \mid p(x)$ label/process of event x



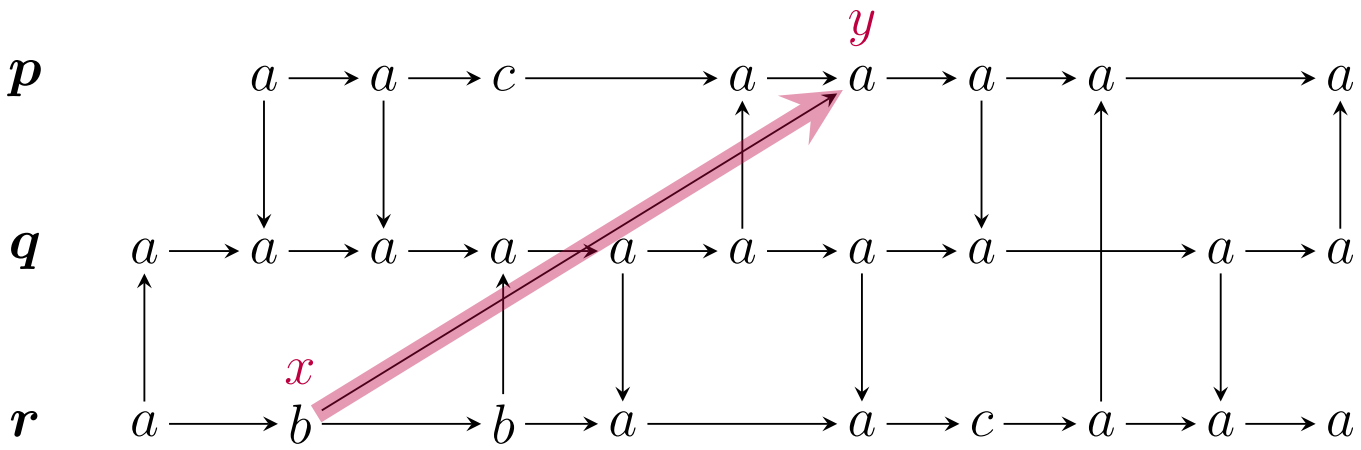
Monadic Second-Order Logic (MSO) over MSCs

$$\varphi ::= \quad a(x) \mid p(x) \qquad \text{label/process of event } x$$
$$\mid x \to y \qquad \text{process successor}$$



Monadic Second-Order Logic (MSO) over MSCs

$$\begin{aligned} \varphi ::= \quad & a(x) \mid p(x) && \text{label/process of event } x \\ \mid\ & x \to y && \text{process successor} \\ \mid\ & x \searrow y && \text{message relation} \end{aligned}$$



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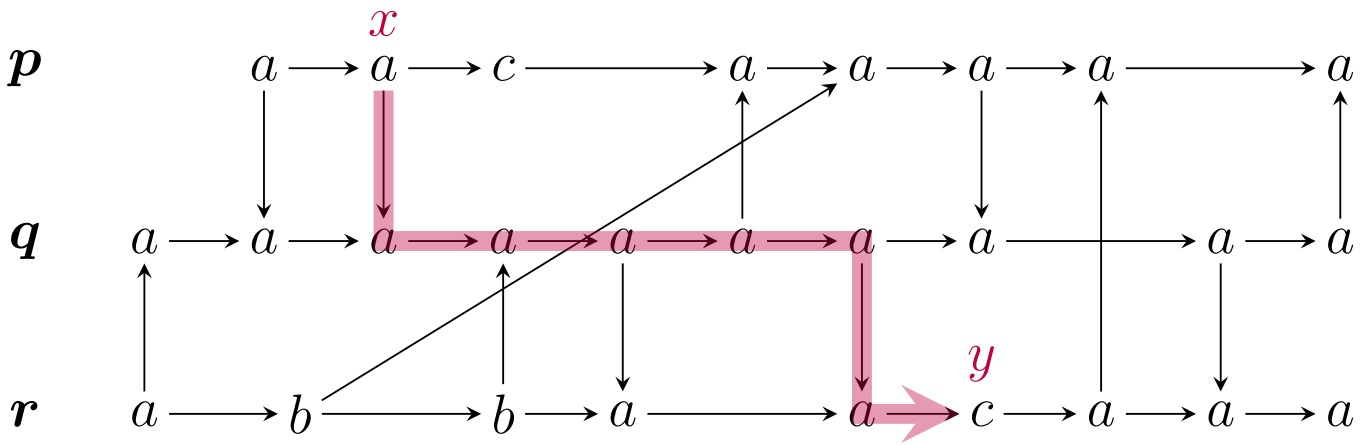
\mid & x \leq y & \text{happened-before}
\end{array}
$$



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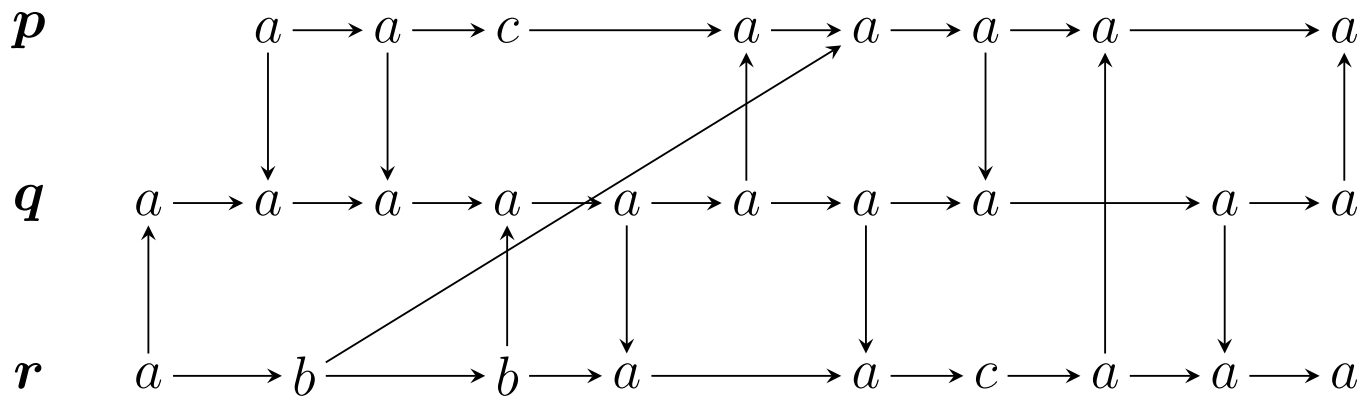
\mid & \neg\varphi \mid \varphi \vee \varphi \mid \exists x.\varphi \mid \exists X.\varphi \mid x \in X &
\end{array}
$$



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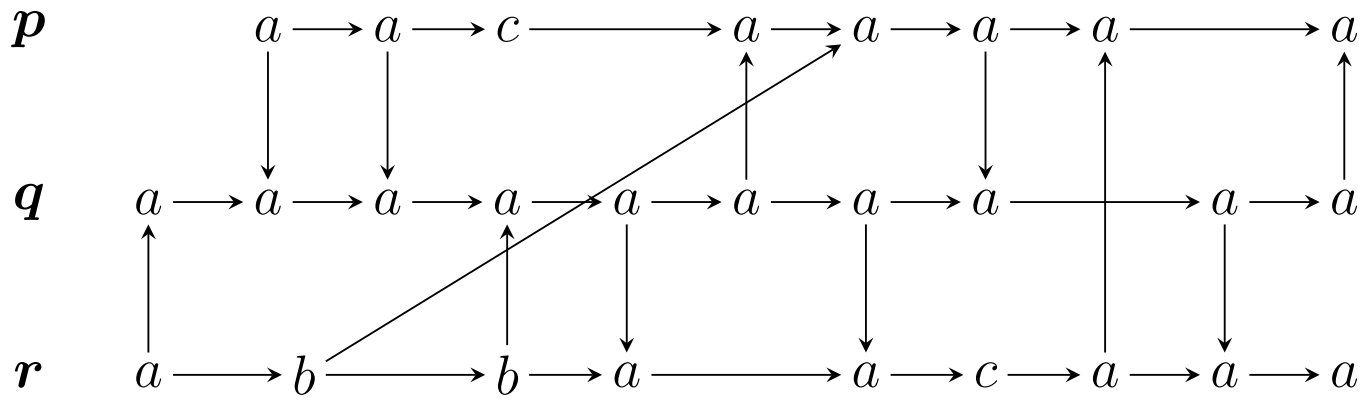

Mutual exclusion: $\neg(\exists x.\exists y.c(x) \wedge c(y) \wedge\ x \parallel y\ )$

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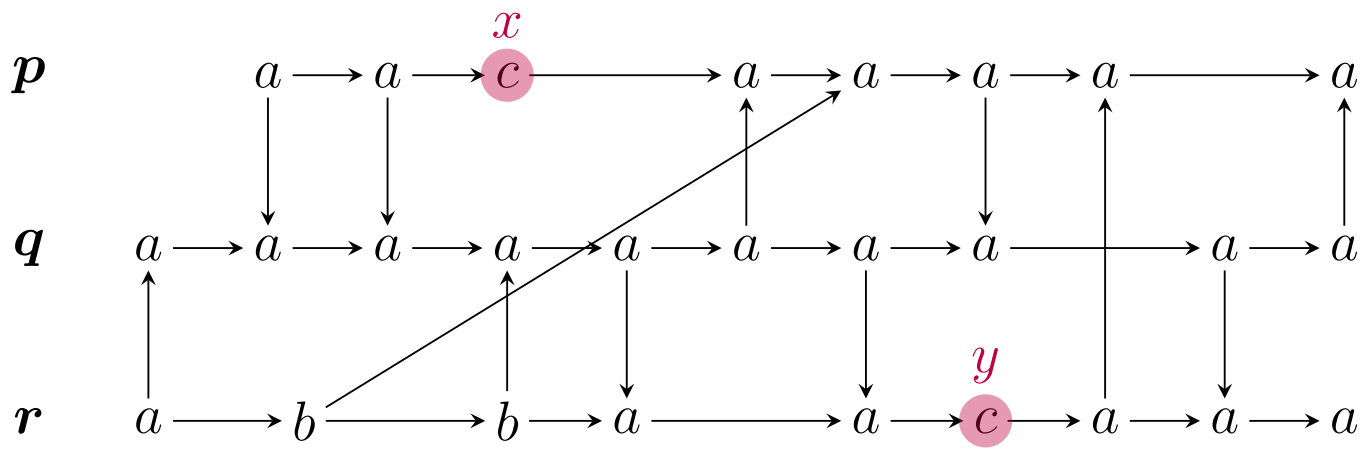

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$\neg(x \leq y) \wedge \neg(y \leq x)$

Büchi-like theorems for CFMs

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When channels are **bounded**:

Büchi-like theorems for CFMs

When channels are **bounded**:

Theorem (Henriksen-Mukund-Narayan Kumar-Sohoni-Thiagarajan '05)

Over universally bounded MSCs, CFM $=$ MSO$[\searrow, \rightarrow, \leq]$.

Theorem (Genest-Kuske-Muscholl '06)

Over existentially bounded MSCs, CFM $=$ MSO$[\searrow, \rightarrow, \leq]$.

General case

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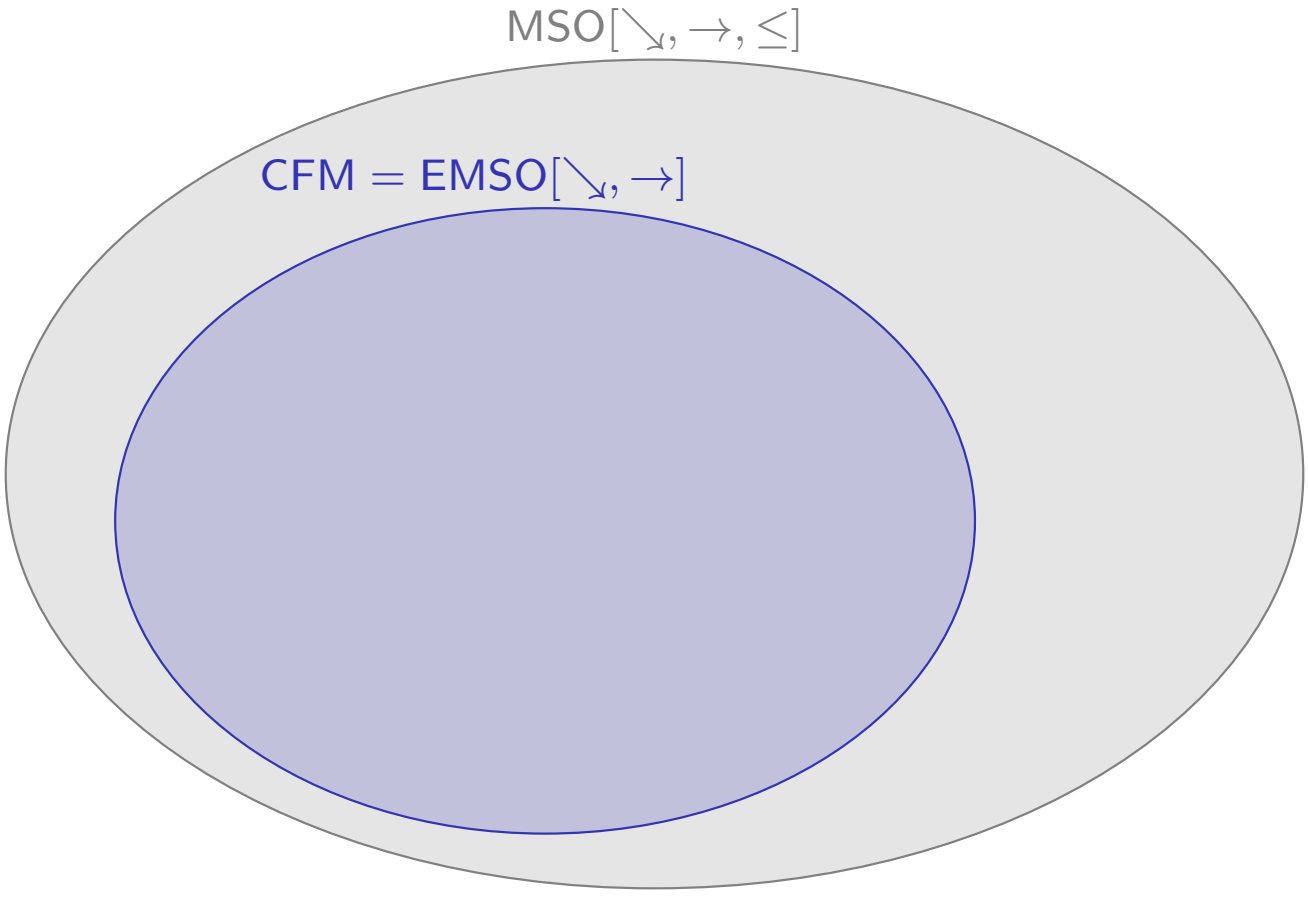
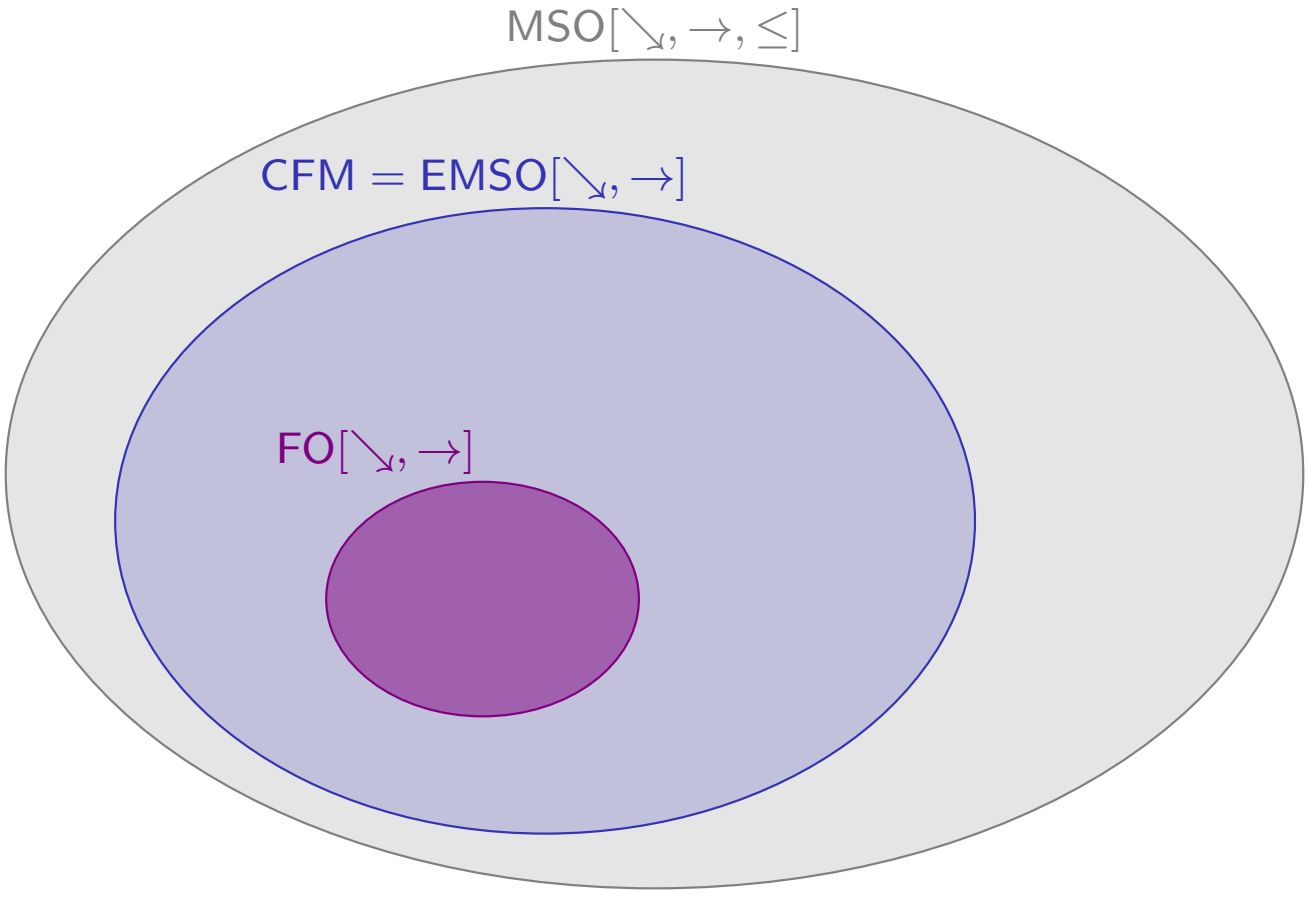


[B.-Leucker '06] $CFM = EMSO[\searrow, \rightarrow] \subsetneq MSO[\searrow, \rightarrow]$

General case



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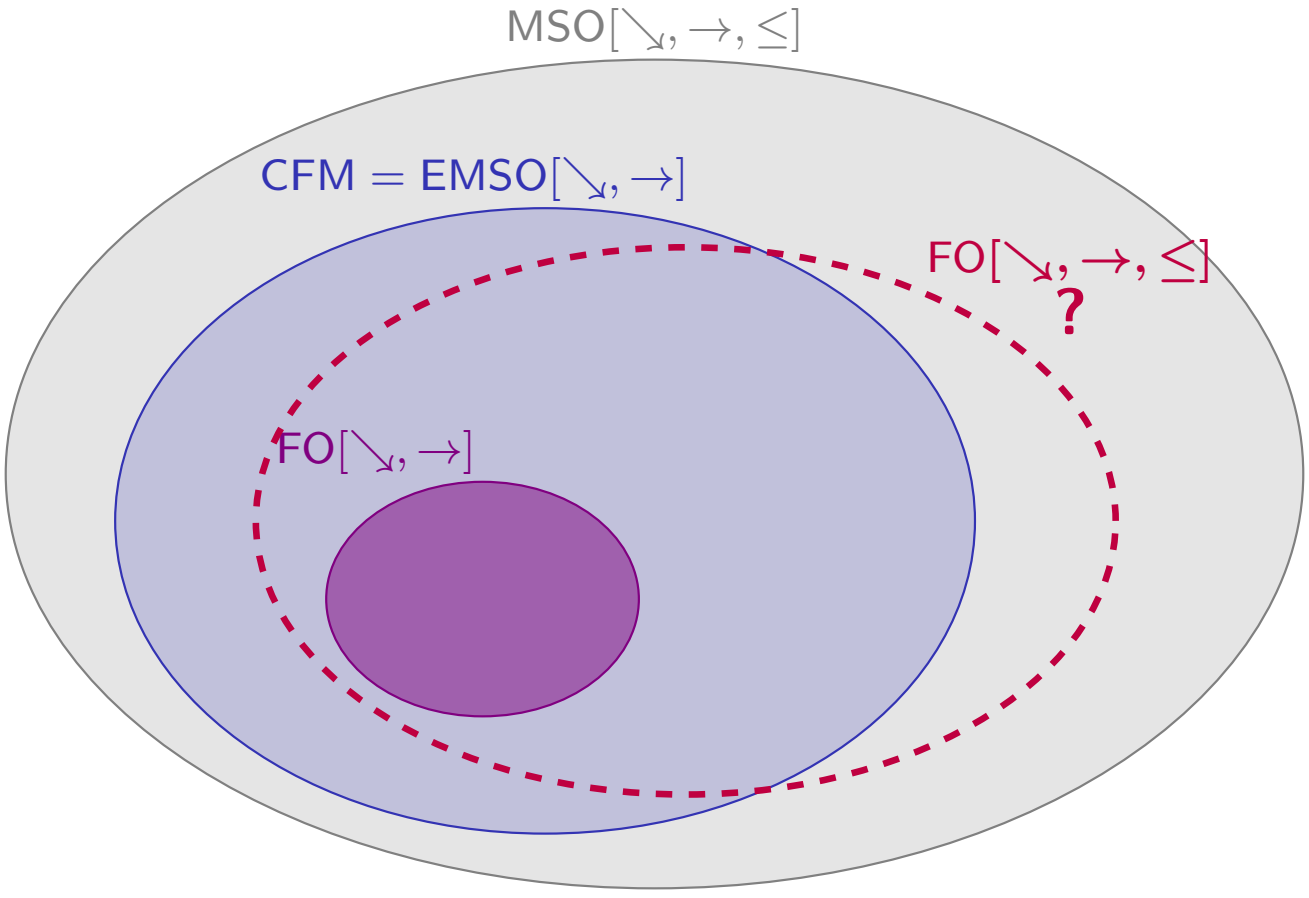

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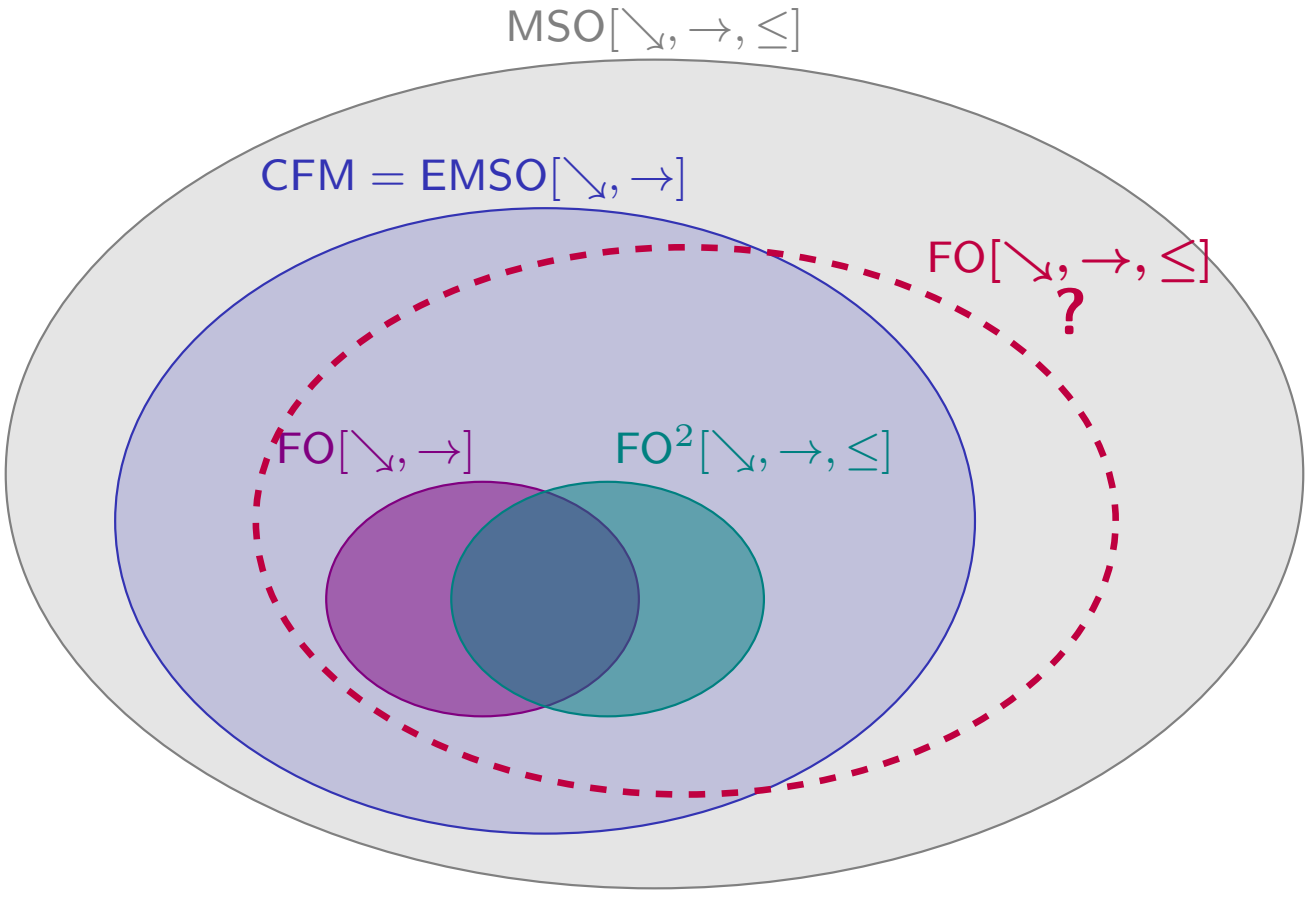

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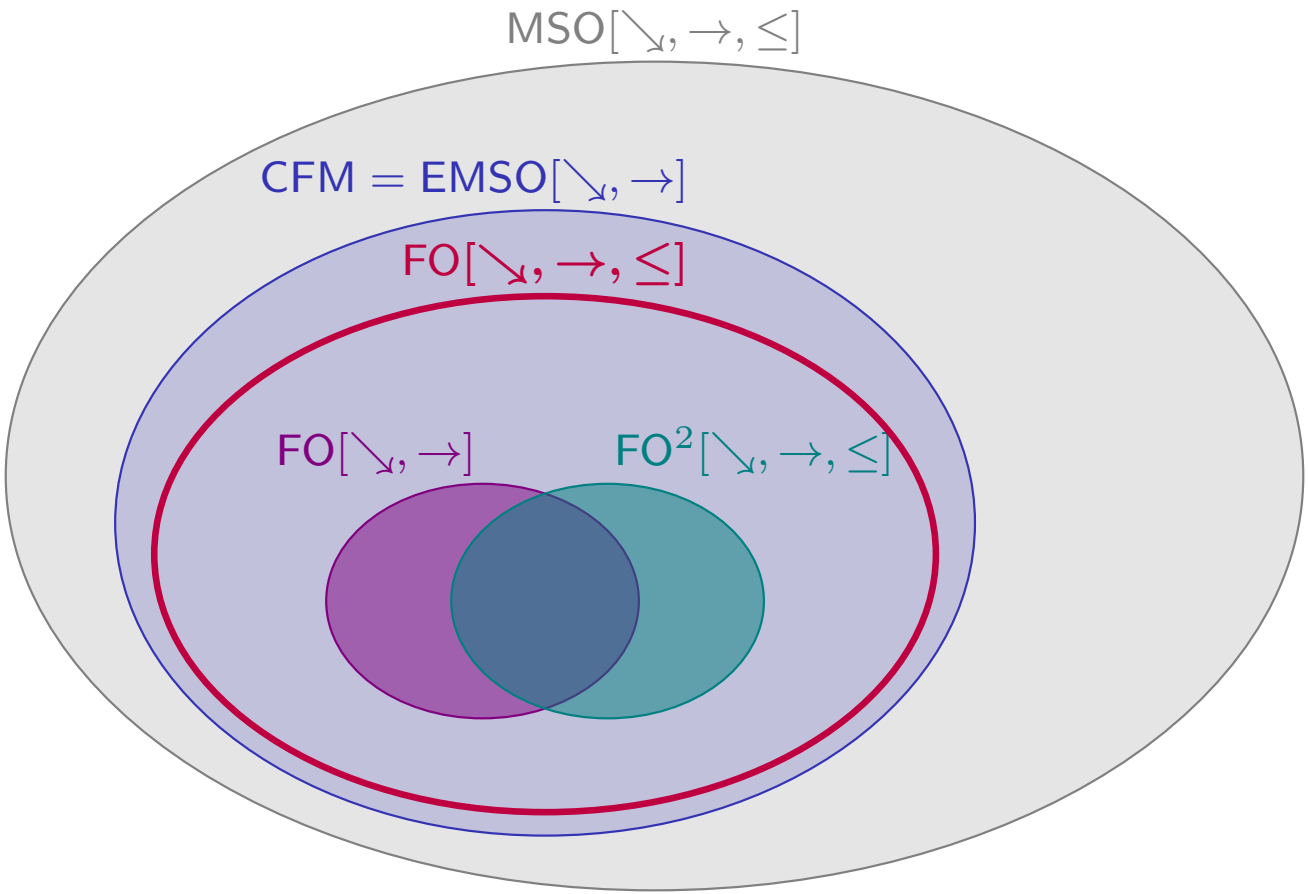
[B.-Fortin-Gastin '18] $\mathsf{CFM} = \mathsf{EMSO}^2[\searrow, \rightarrow, \leq]$

General case



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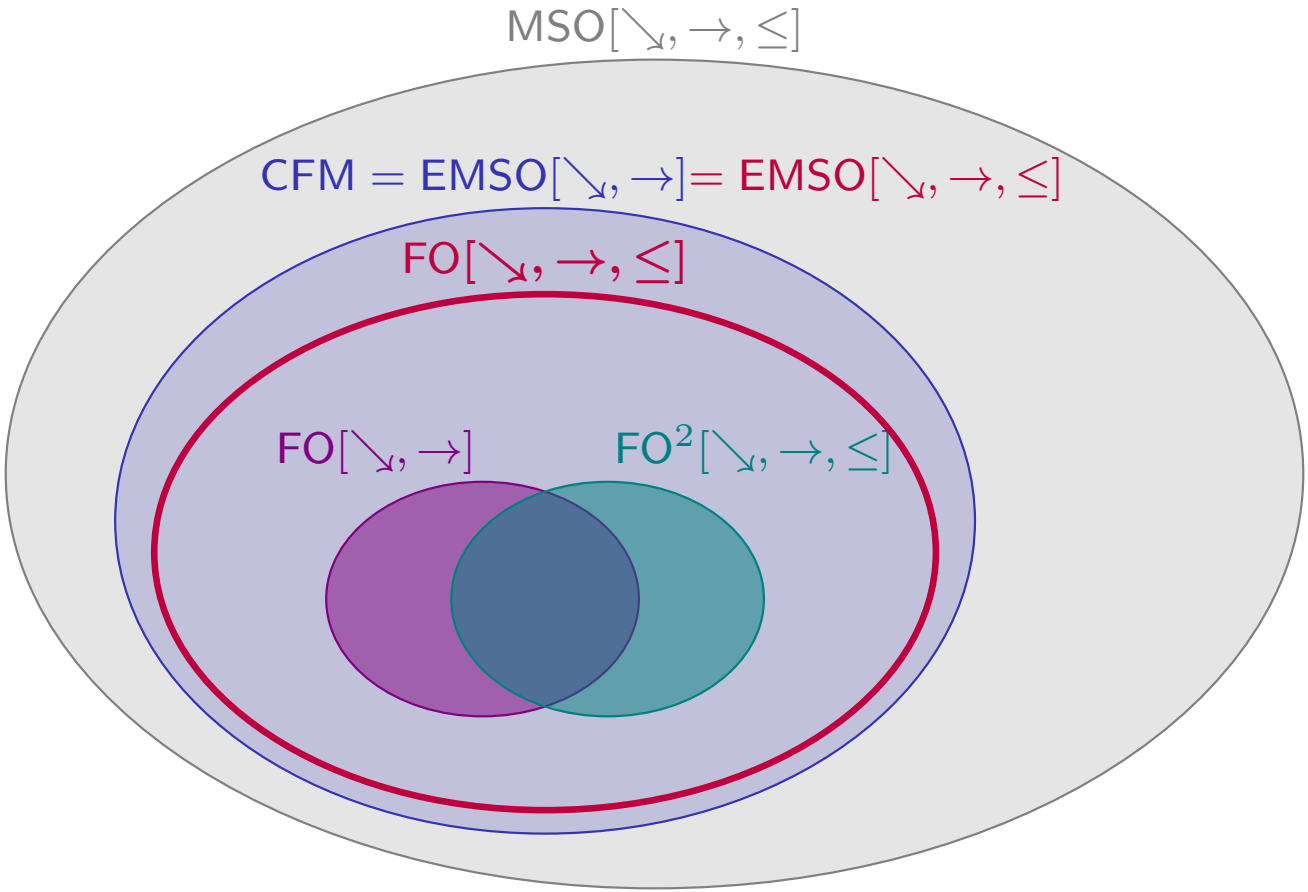
[B.-Fortin-Gastin '18] $\mathsf{CFM} = \mathsf{EMSO}^2[\searrow, \rightarrow, \leq]$

Main result: $\mathsf{CFM} = \mathsf{EMSO}[\searrow, \rightarrow, \leq]$

General case



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Main result: $CFM = EMSO[\searrow, \rightarrow, \leq]$

Translation from FO to CFMs

Goal

$FO[\searrow, \rightarrow, \leq]$ sentence φ

CFM $\mathcal{A}$ such that $L(\mathcal{A}) = L(\varphi)$

Translation from FO to CFMs

Goal

- CFMs are not closed under complementation
  – no direct induction on φ

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$FO[\searrow, \rightarrow, \leq]$ sentence φ → CFM $\mathcal{A}$ such that $L(\mathcal{A}) = L(\varphi)$

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Translation from FO to CFMs

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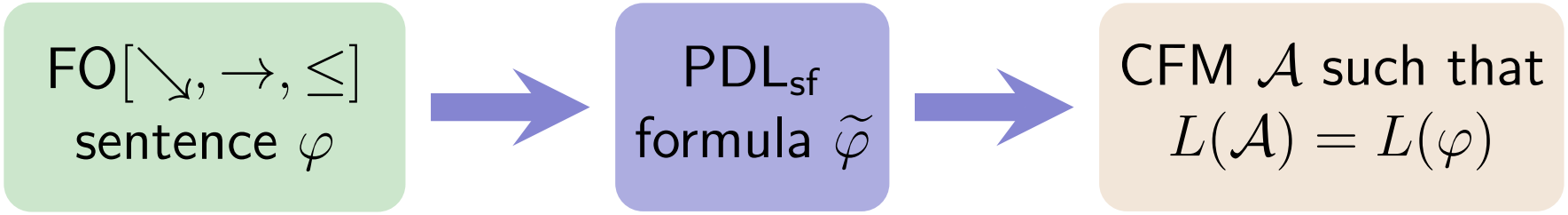

- CFMs are not closed under complementation
  – no direct induction on φ
- Techniques used for previous cases do not apply here

Solution: go through an intermediate language:
"Star-free" Propositional Dynamic Logic (with Loop and Converse)

A simple modal logic for MSCs: PDL_{sf}^{-}

$\varphi, \psi ::=$

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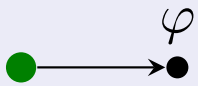

("Next φ")

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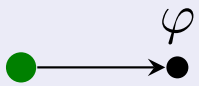

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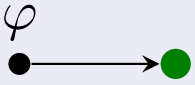

("Yesterday φ")

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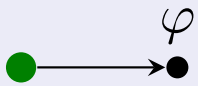

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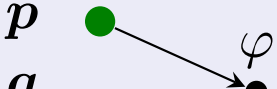

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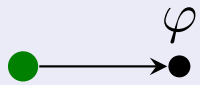

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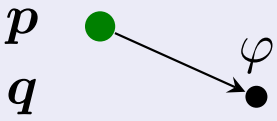
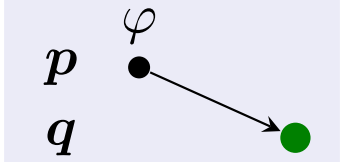
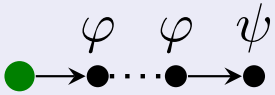




$$\mid \langle \stackrel{\varphi}{\rightarrow} \rangle \psi$$



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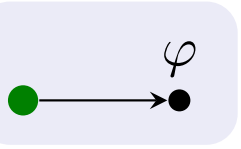

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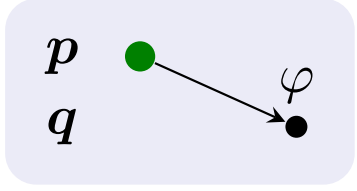

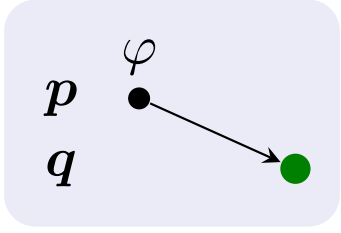

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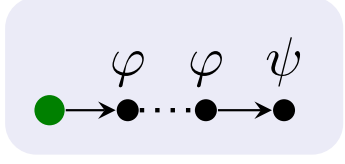

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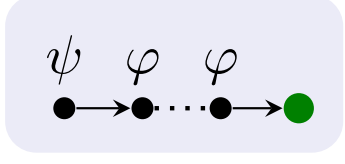

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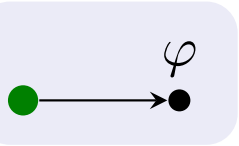

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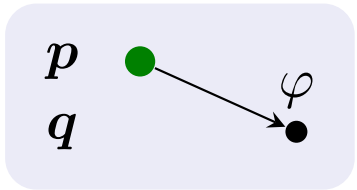

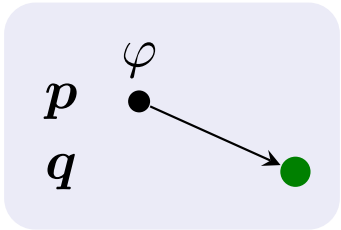

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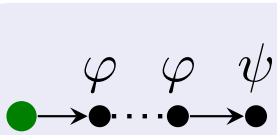

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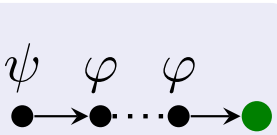
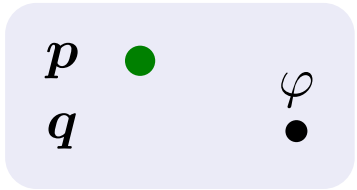


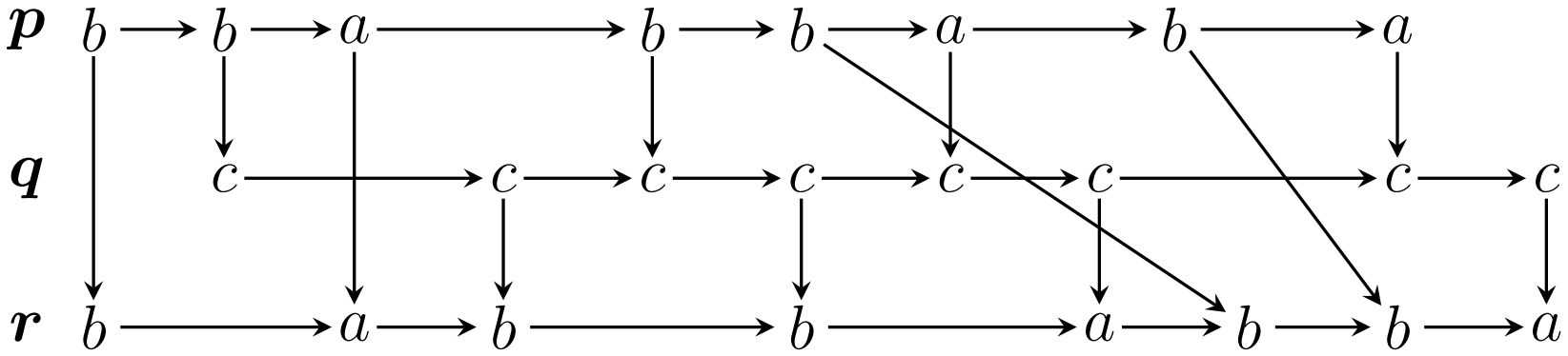
("φ Since ψ")

$$\mid \langle \mathsf{jump}_{p,q} \rangle \varphi$$



Examples

$$P = \{p, q, r\},\ \Sigma = \{a, b, c\}$$



"On q, a receive from p is directly followed by a send to r":

$$\langle {}^{p}\!\nwarrow_{q} \rangle\, true \implies \langle \rightarrow \rangle \langle {}^{q}\!\searrow_{r} \rangle\, true$$

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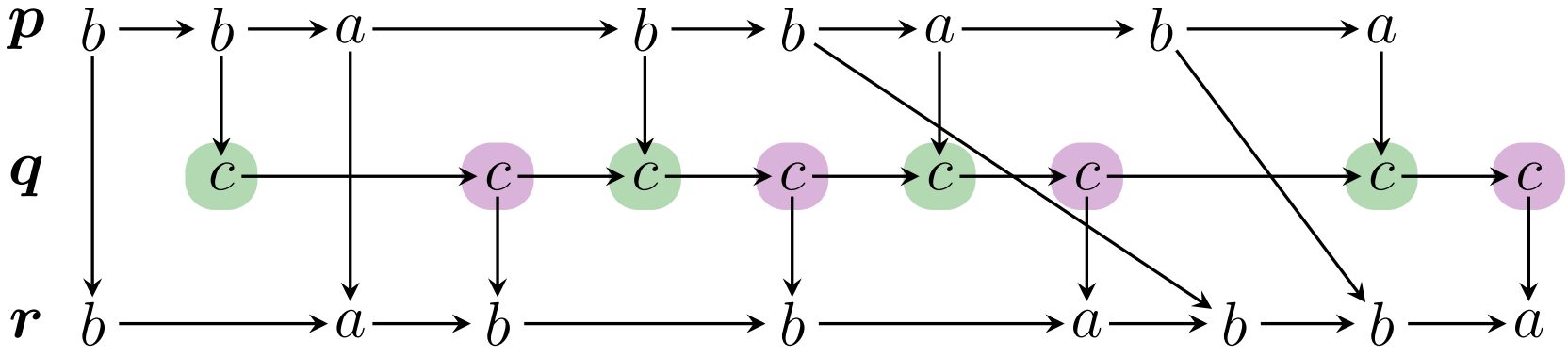

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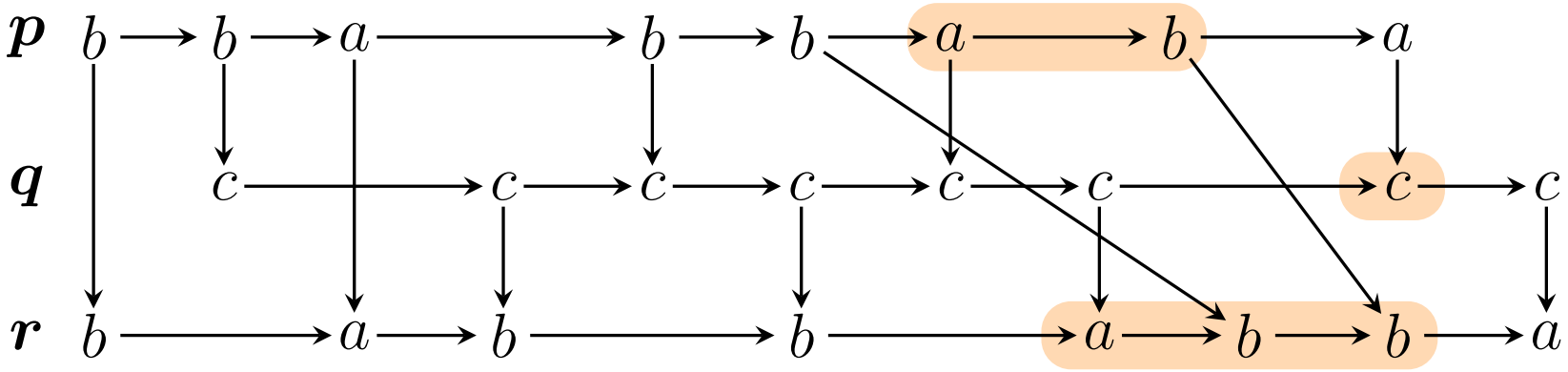

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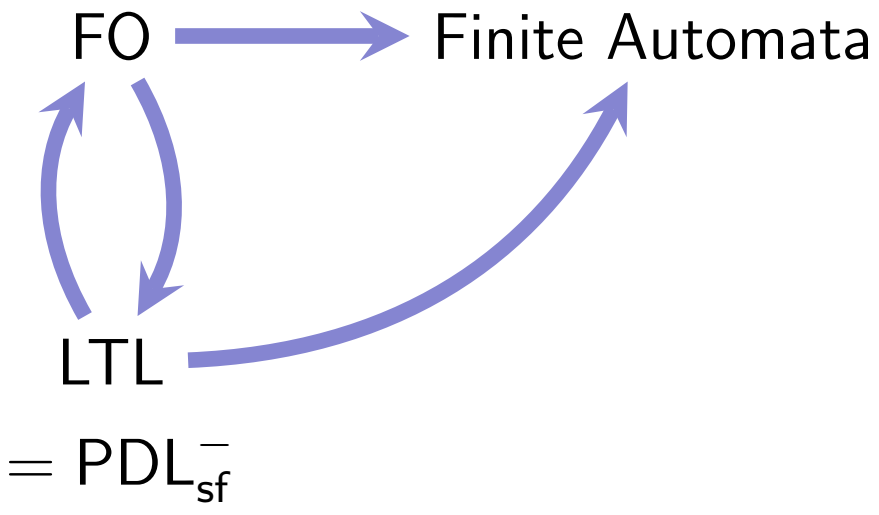
"All events strictly in between the current and the last event on a process are bs":

$$\langle \overset{b}{\rightarrow} \rangle \neg \langle \rightarrow \rangle\, true$$

Expressivity of PDL_{sf}^{-}

Over words ($|P| = 1$):



$= PDL_{sf}^{-}$

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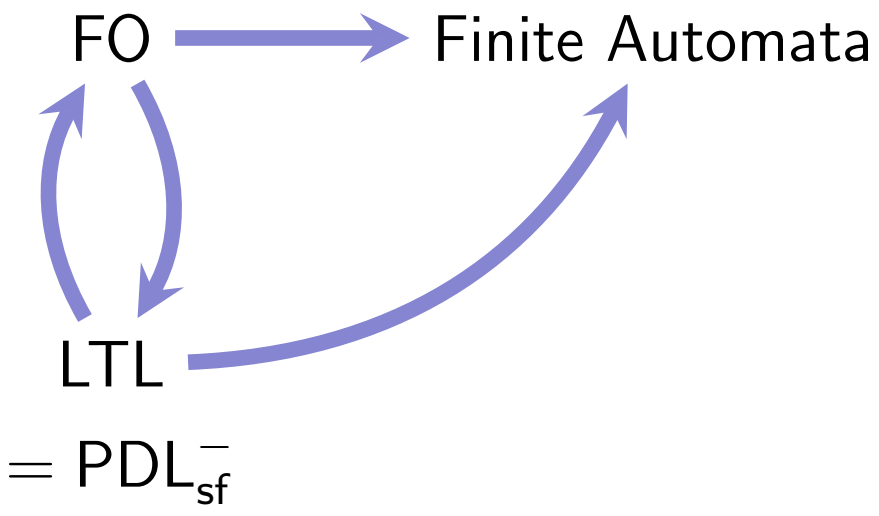
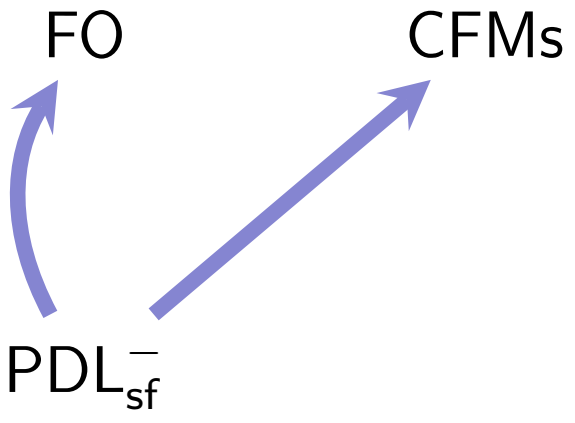


Over general MSCs:



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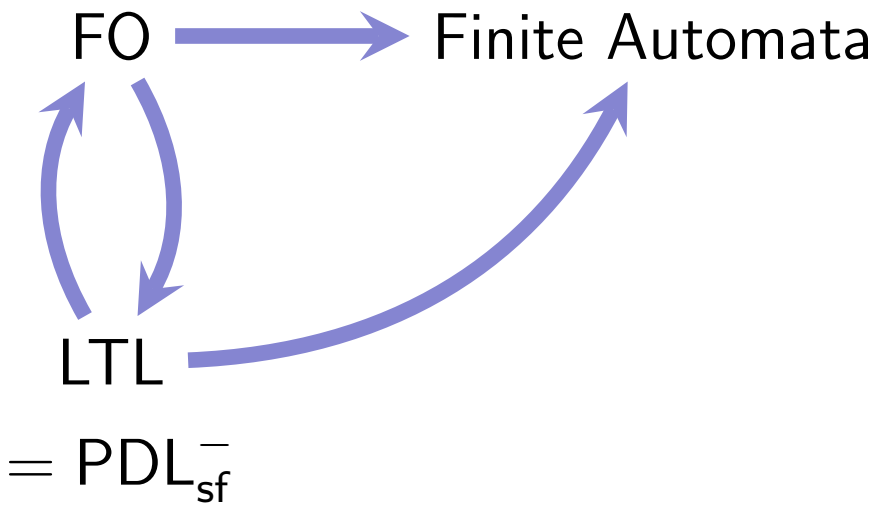

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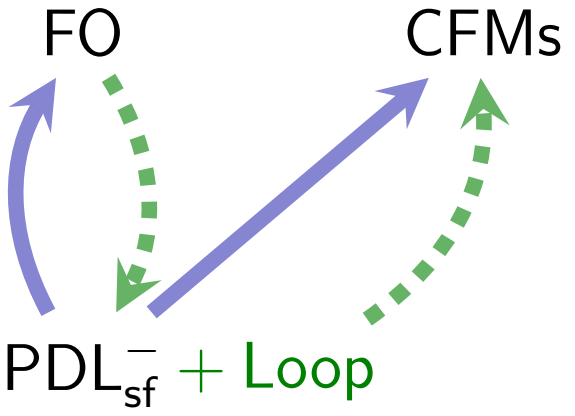

Star-free Propositional Dynamic Logic (PDL_{sf})

[Fischer-Ladner 1979] (PDL)

Event formulas

$$\varphi ::= a \mid p \mid \varphi \vee \varphi \mid \neg\varphi$$

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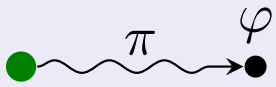

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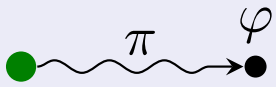

$$\pi ::= \rightarrow \mid \leftarrow \mid {}^{p}\searrow_{q} \mid {}^{p}\nwarrow_{q} \mid \mathsf{jump}_{p,q} \mid \overset{\varphi}{\rightarrow} \mid \overset{\varphi}{\leftarrow}$$

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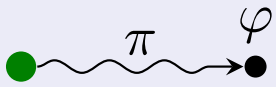

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Notation: $\langle \pi_1 \cdot \pi_2 \cdots \pi_k \rangle \varphi \equiv \langle \pi_1 \rangle (\langle \pi_2 \rangle \cdots (\langle \pi_k \rangle \varphi) \cdots)$

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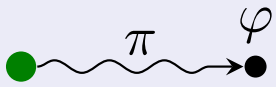

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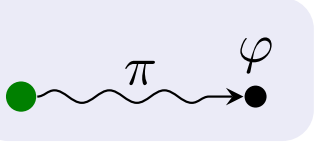

Path formulas

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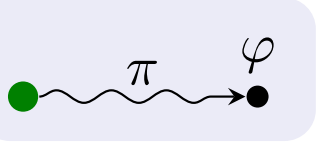

$$\mid \mathsf{Loop}(\pi)$$

Path formulas

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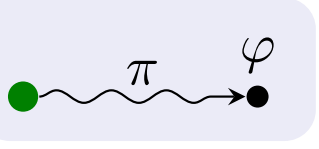

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$$\mid \pi \cup \pi \mid \pi \cap \pi \mid \pi^{c}$$

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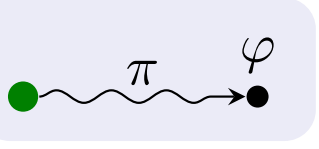


$$\mid \mathsf{Loop}(\pi)$$

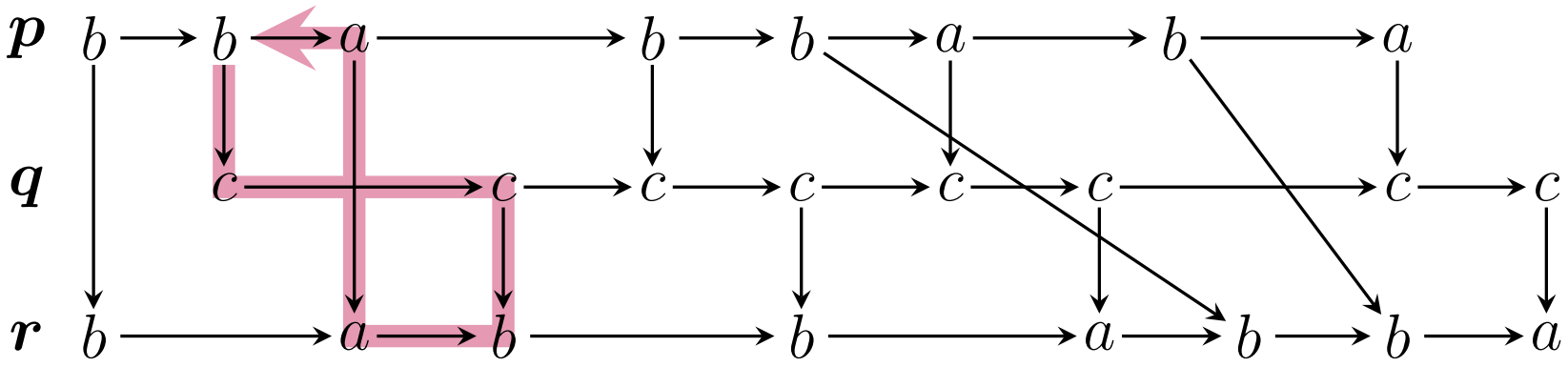
Path formulas

$$\pi ::= \rightarrow \mid \leftarrow \mid {}^{p}\searrow_{q} \mid {}^{p}\nwarrow_{q} \mid \mathsf{jump}_{p,q} \mid \xrightarrow{\varphi} \mid \xleftarrow{\varphi} \mid \pi \cdot \pi \mid \{\varphi\}?$$

Notation: $\langle \pi_1 \cdot \pi_2 \cdots \pi_k \rangle \varphi \equiv \langle \pi_1 \rangle (\langle \pi_2 \rangle \cdots (\langle \pi_k \rangle \varphi) \cdots)$

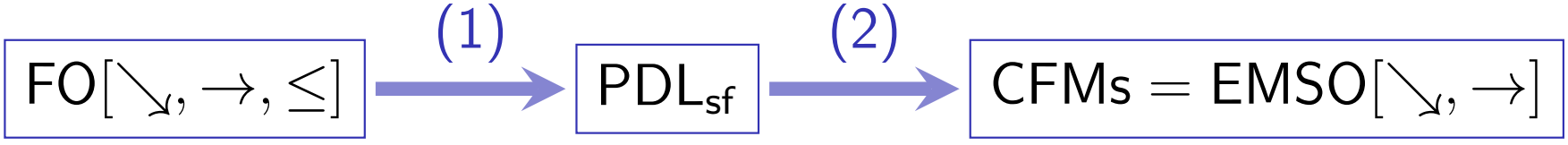
Example

$$P = \{p, q, r\},\ \Sigma = \{a, b, c\}$$



$$\varphi = \mathsf{Loop}({}^{p}\!\searrow_{q} \cdot \rightarrow \cdot {}^{q}\!\searrow_{r} \cdot \{b\}? \cdot \leftarrow \cdot {}^{p}\!\nwarrow_{r} \cdot \leftarrow)$$

Main results



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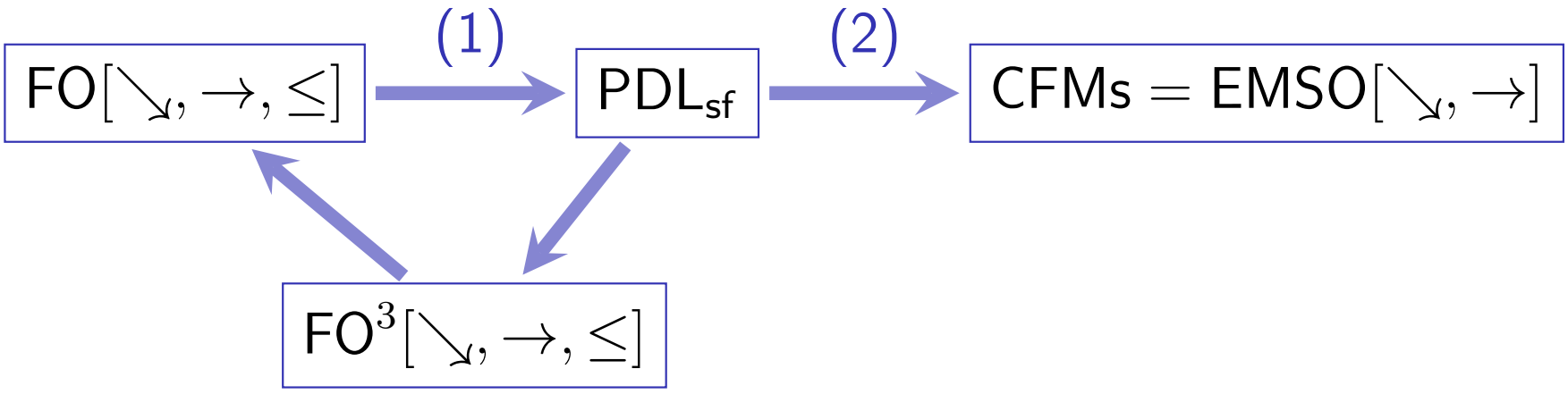

From PDL_{sf} to FO^3

- Any PDL_{sf} event formula φ can be transformed into an FO^3 formula $\widetilde{\varphi}(x)$ with one free variable.

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$$p \quad \rightsquigarrow \quad p(x)$$

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- Any $\mathsf{PDL}_{\mathsf{sf}}$ event formula φ can be transformed into an FO^3 formula $\widetilde{\varphi}(x)$ with one free variable.

$$\begin{array}{rcl} p & \rightsquigarrow & p(x) \\ \langle \pi \rangle \, \varphi & \rightsquigarrow & \big(\exists y. \widetilde{\varphi}(y) \wedge \widetilde{\pi}(x,y)\big)(x) \end{array}$$

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From FO to $\mathsf{PDL}_{\mathsf{sf}}$

Theorem

Any FO formula $\Phi(x_1, \ldots, x_n)$ can be rewritten as

$$\Phi(x_1, \ldots, x_n) \equiv \bigvee \bigwedge \pi(x_i, x_j) \quad \text{where } \pi \in \mathsf{PDL}_{\mathsf{sf}}$$

Proof: By induction.

From FO to PDL_{sf}

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Any FO formula $\Phi(x_1)$ can be rewritten as

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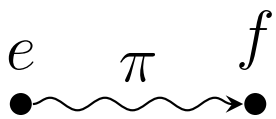
Proof: By induction. Two interesting cases:

- negation
- existential quantification

Key Lemma

For all $\pi \in \mathsf{PDL}_{\mathsf{sf}}$,



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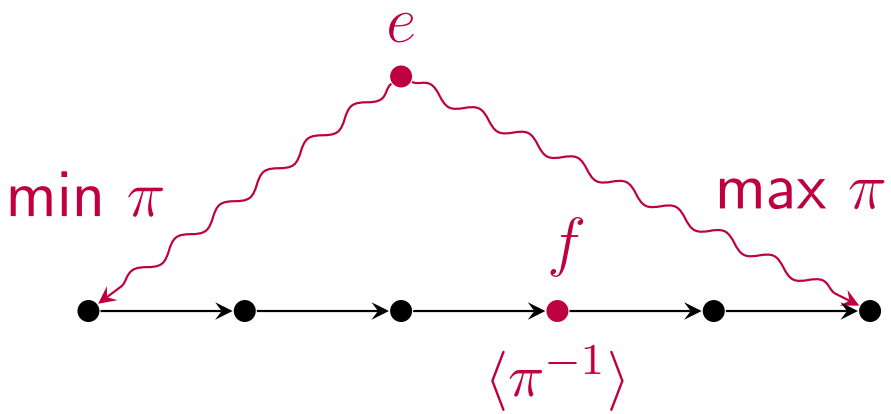

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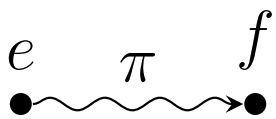

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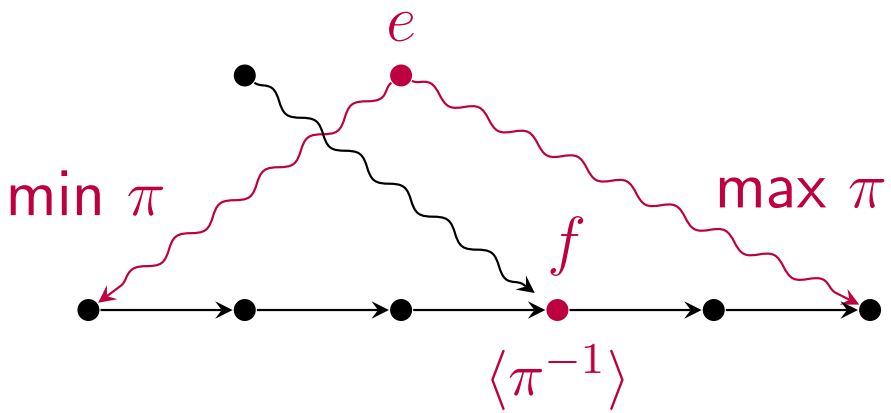

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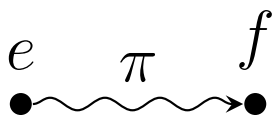

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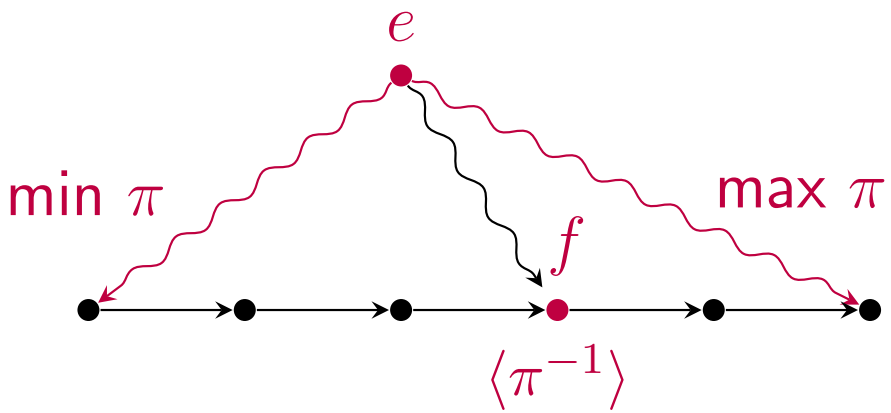

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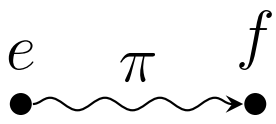

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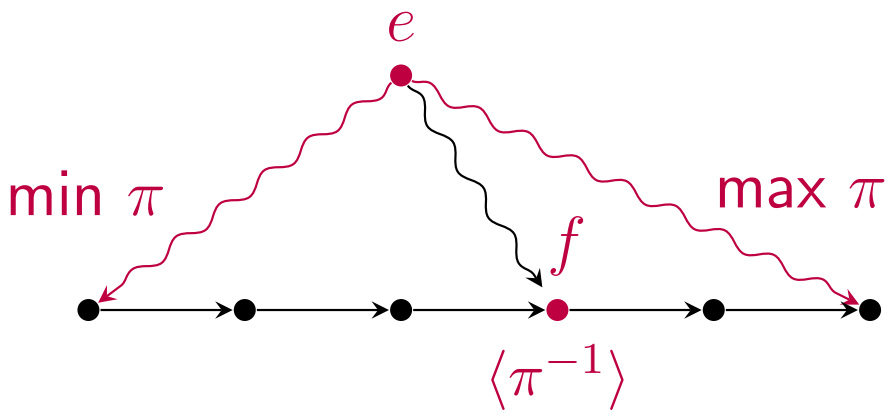

- Monotonicity of path formulas.
- Relies on FIFO behavior.
- Proof is by simple induction.

Negation

$$\neg \bigvee \bigwedge \pi(x_i, x_j) \equiv \bigwedge \bigvee \neg\pi(x_i, x_j)$$

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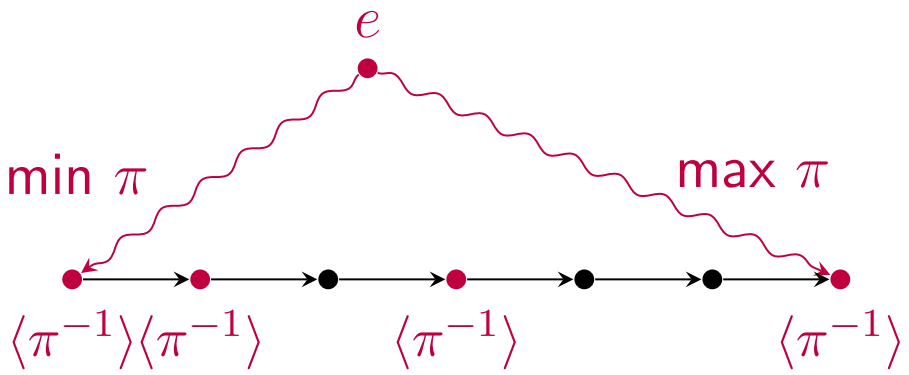

$\pi^{\mathsf{c}} \equiv$

Negation

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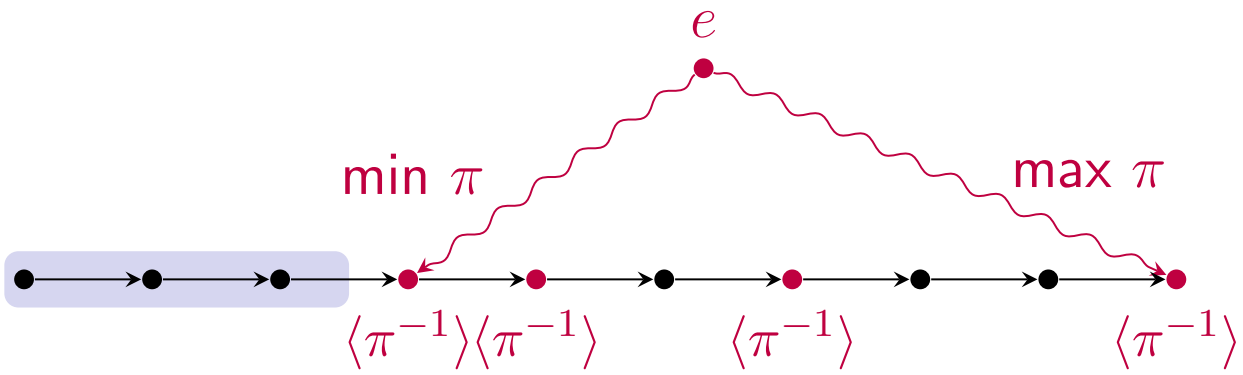

$\pi^{c} \equiv$ min $\pi \cdot \overleftarrow{+}$

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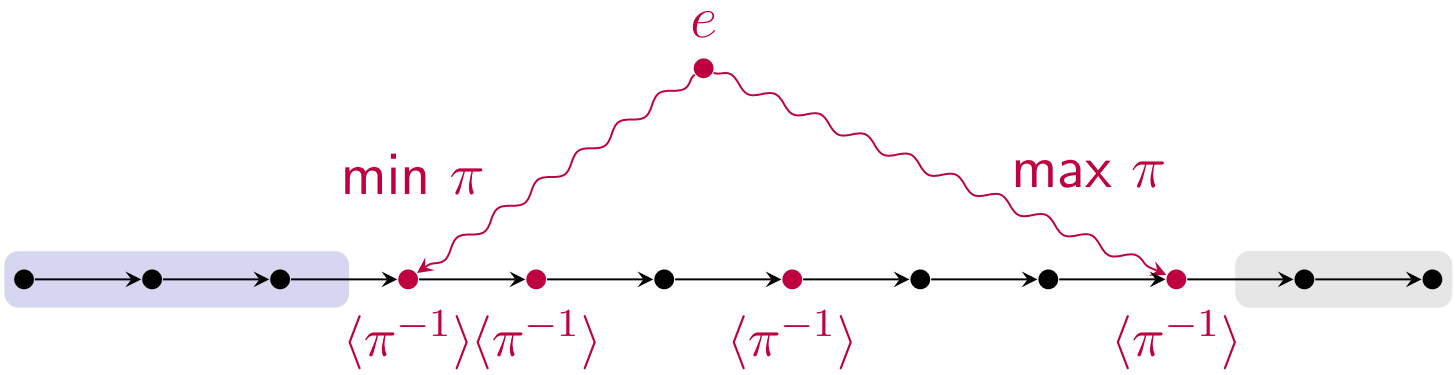

$$\pi^{c} \equiv \text{min } \pi \cdot \overleftarrow{+} \cup \text{max } \pi \cdot \overrightarrow{+}$$

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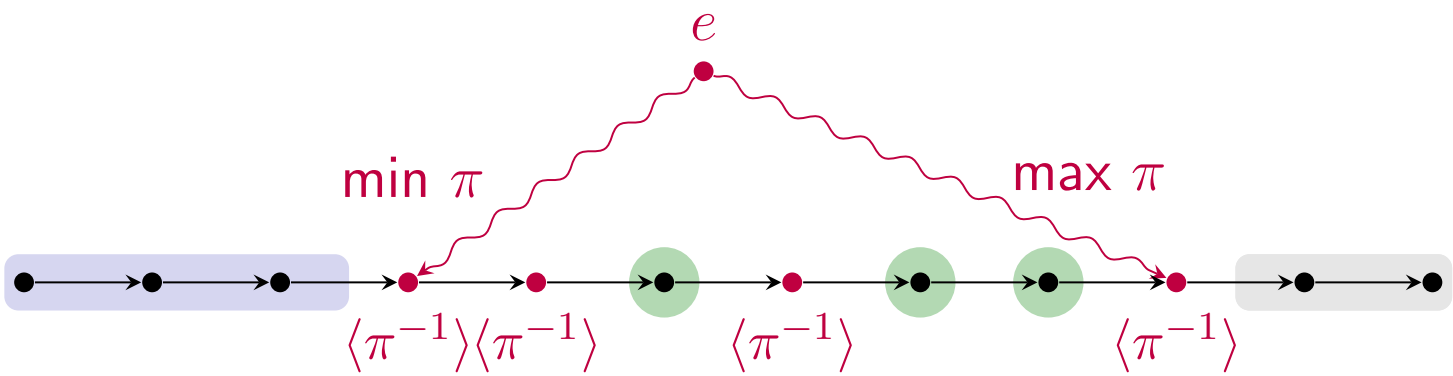

$$\pi^{\mathsf{c}} \equiv \min \pi \cdot \overleftarrow{+} \cup \max \pi \cdot \overrightarrow{+} \cup \min \pi \cdot \overrightarrow{+} \cdot \{\neg \langle \pi^{-1} \rangle\}?$$

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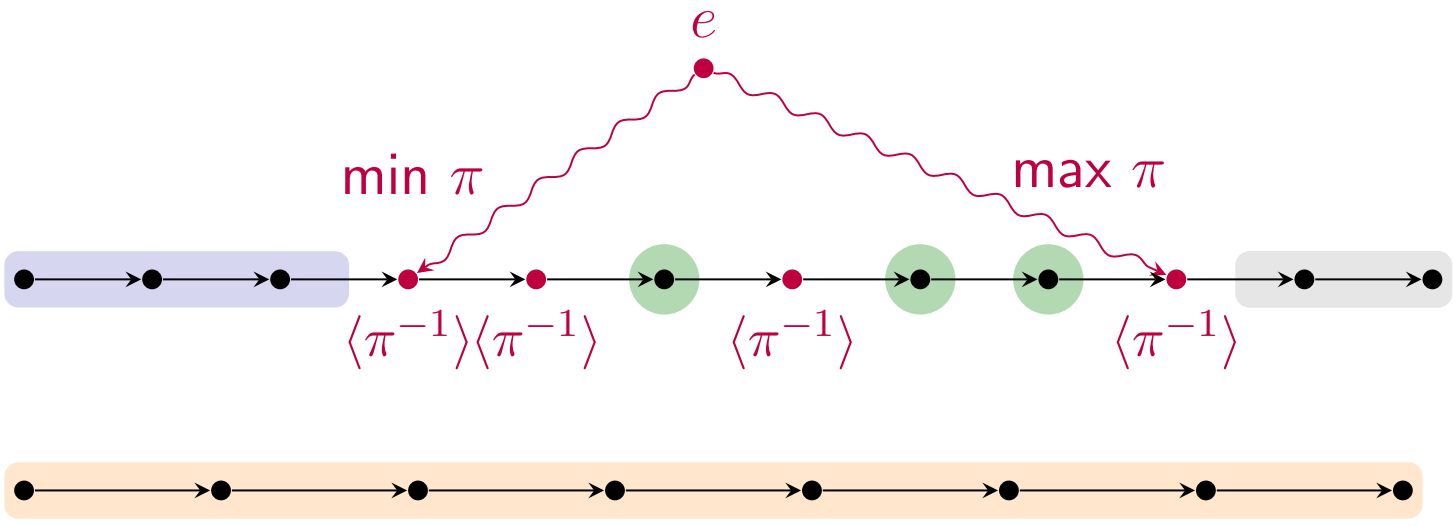

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$$\cup \bigcup_{p,q} \{\neg \langle\pi\rangle\, q\}? \cdot \mathsf{jump}_{p,q}$$

Existential quantification

$$\exists x.\, \bigwedge_i \pi_i(x_i, x) \qquad \rightsquigarrow \qquad ?$$

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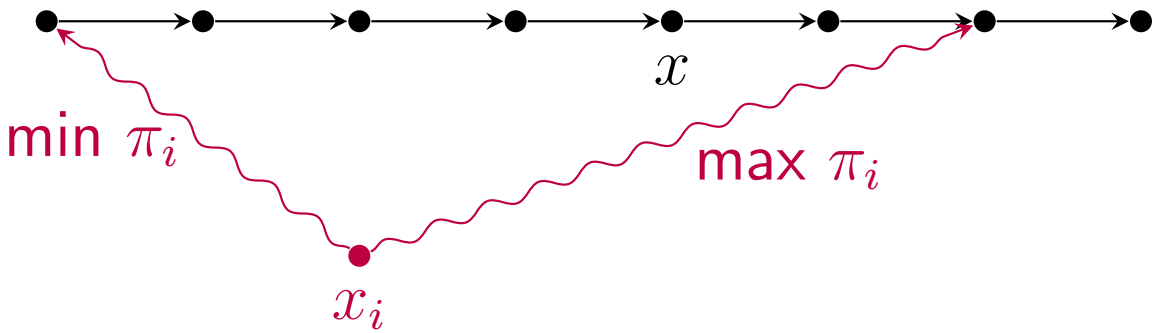

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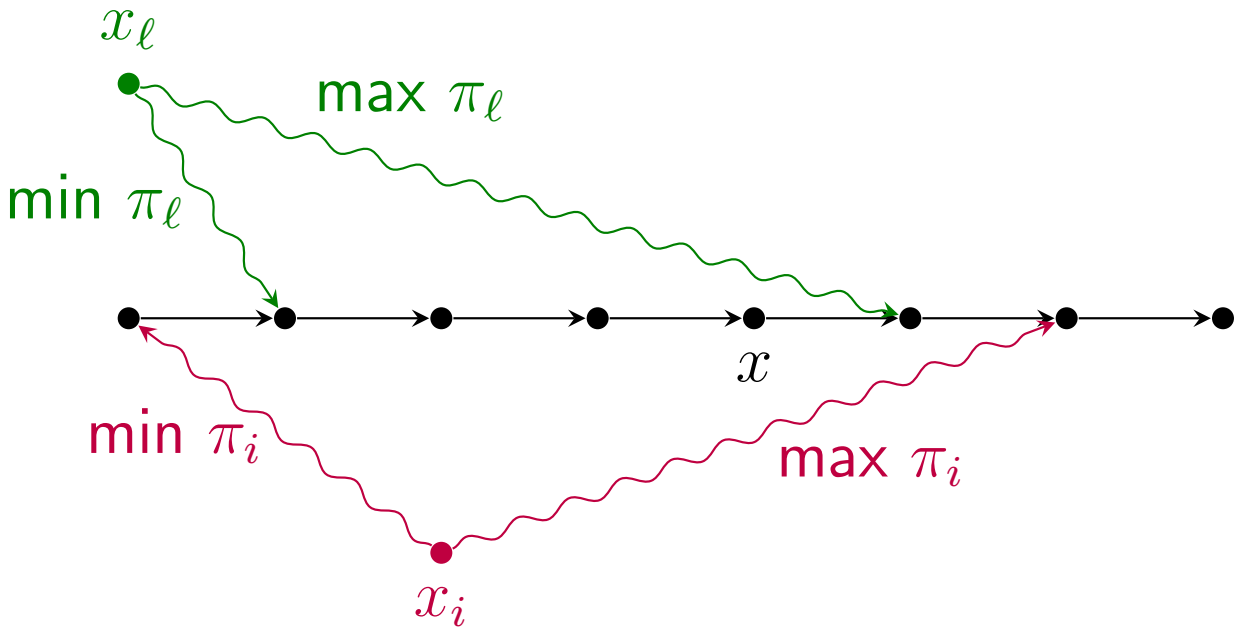

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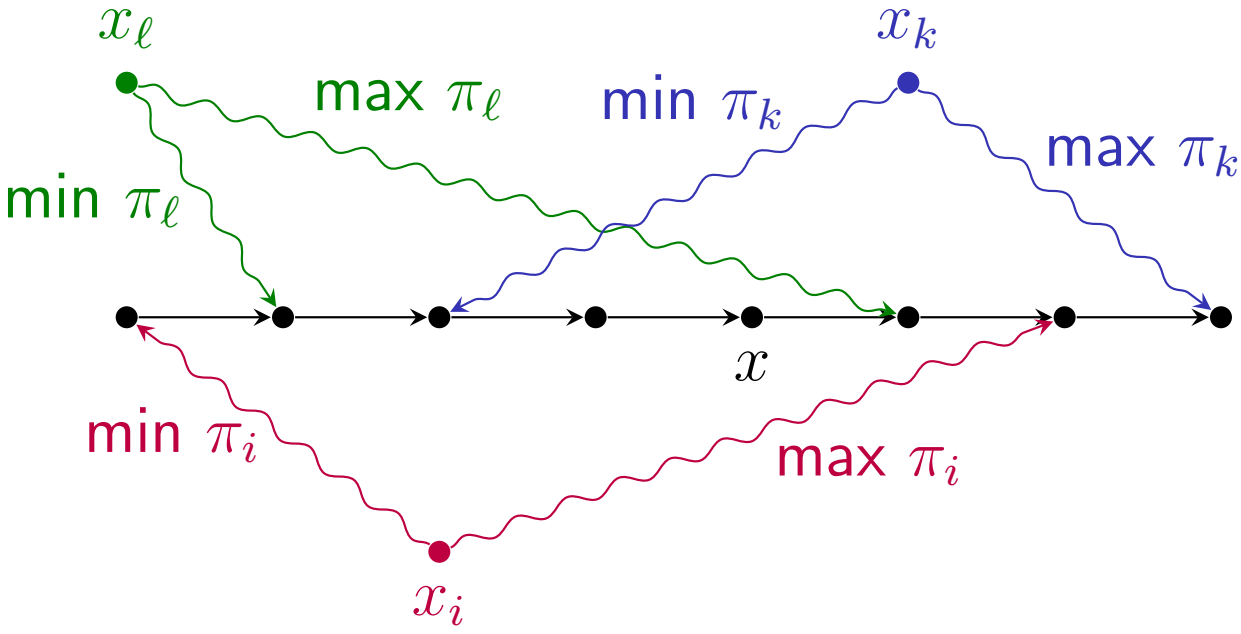

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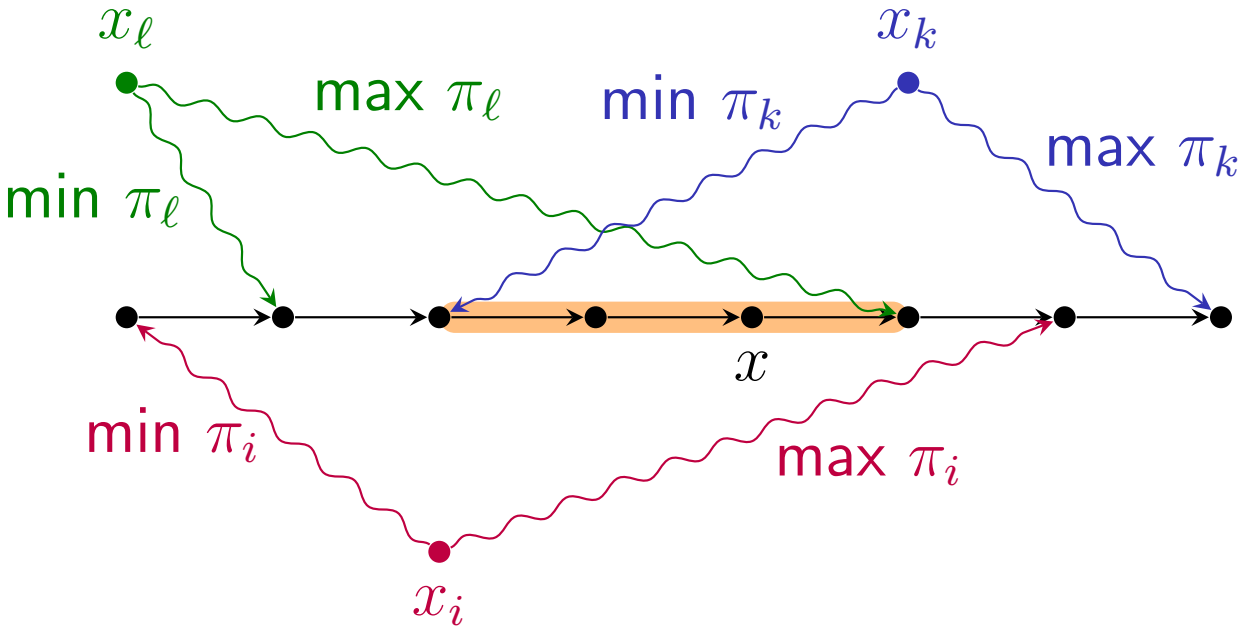

Is there an event in the intersection of the intervals that satisfies $\psi = \bigwedge_i \langle \pi_i^{-1} \rangle$?

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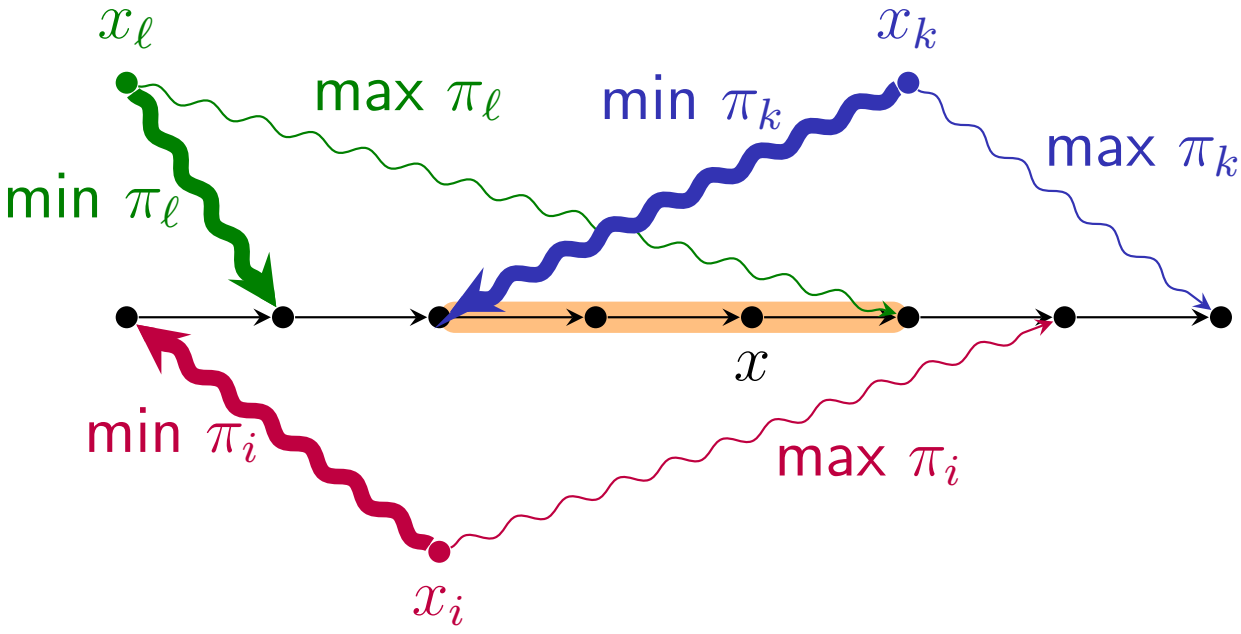

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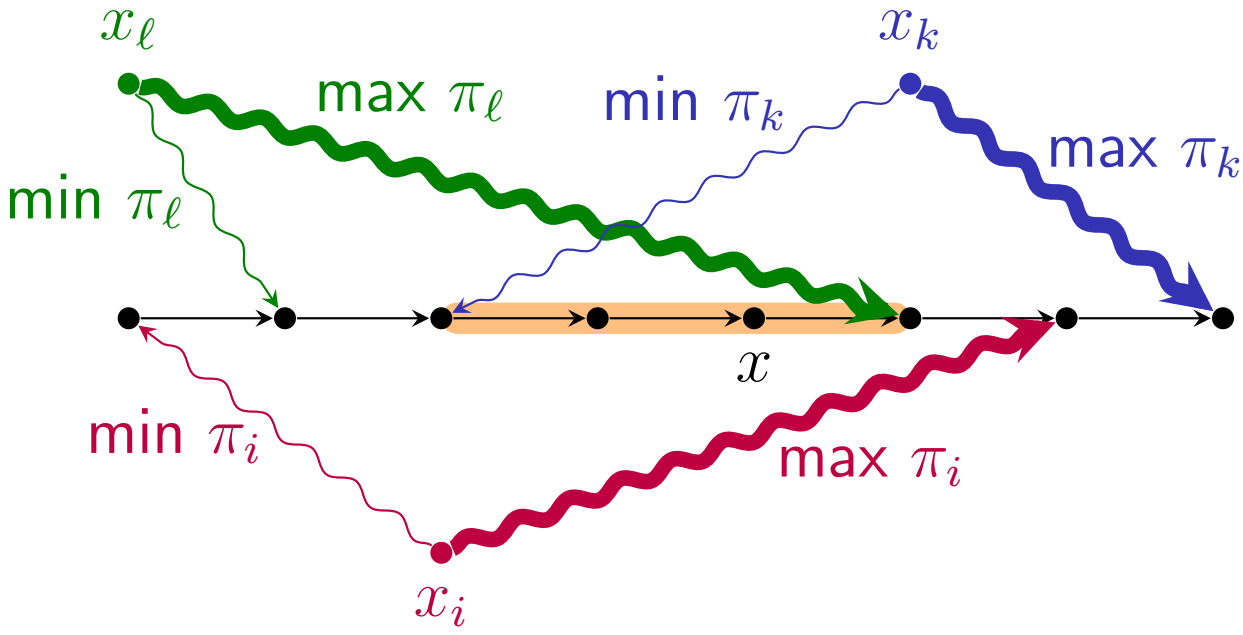

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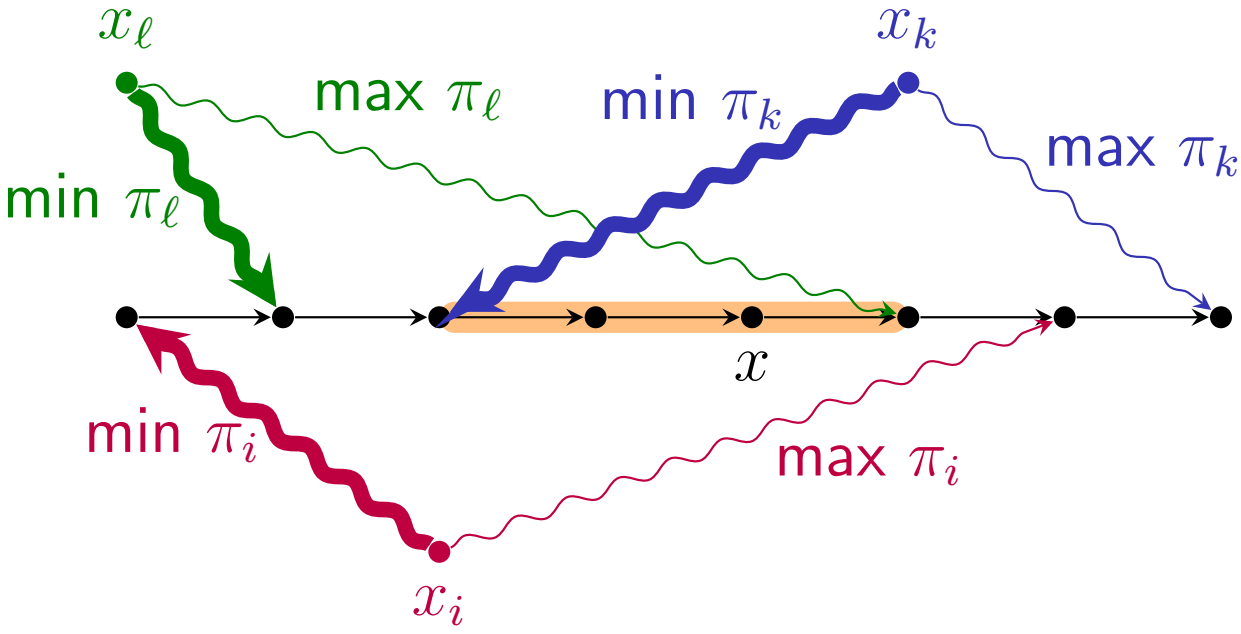

$$\bigvee_k \Big(\qquad \Big)$$

Existential quantification

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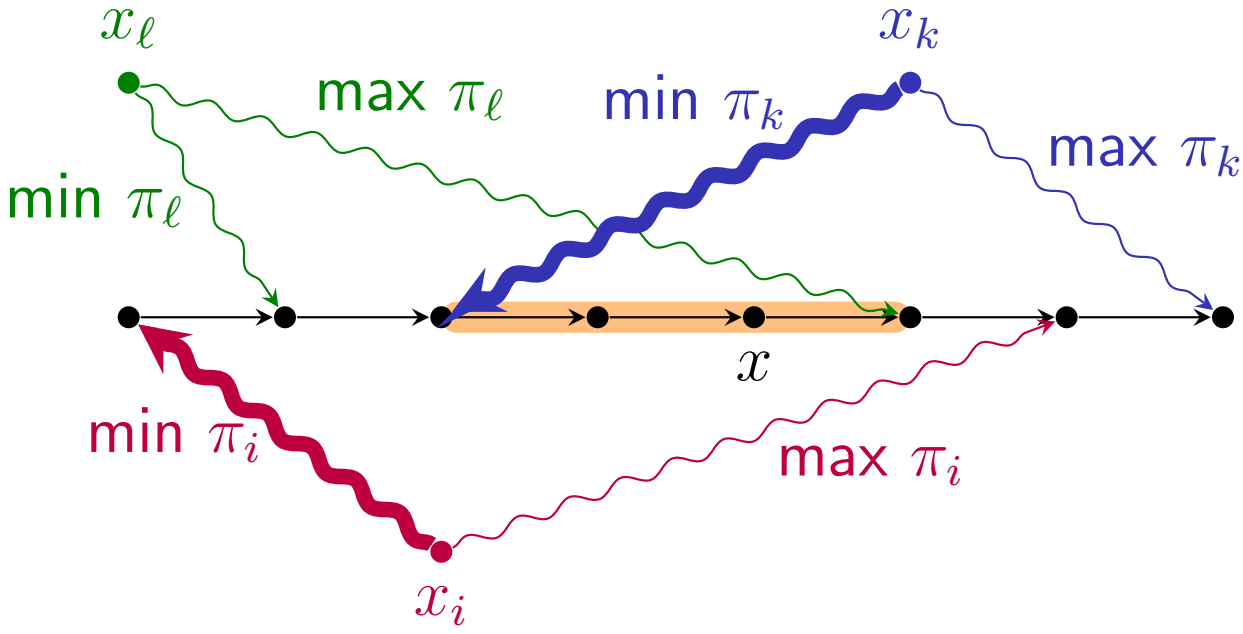

$$\bigvee_k \left(\bigwedge_i (\text{min } \pi_i \cdot \xrightarrow{*} \cdot (\text{min } \pi_k)^{-1})(x_i, x_k) \right)$$

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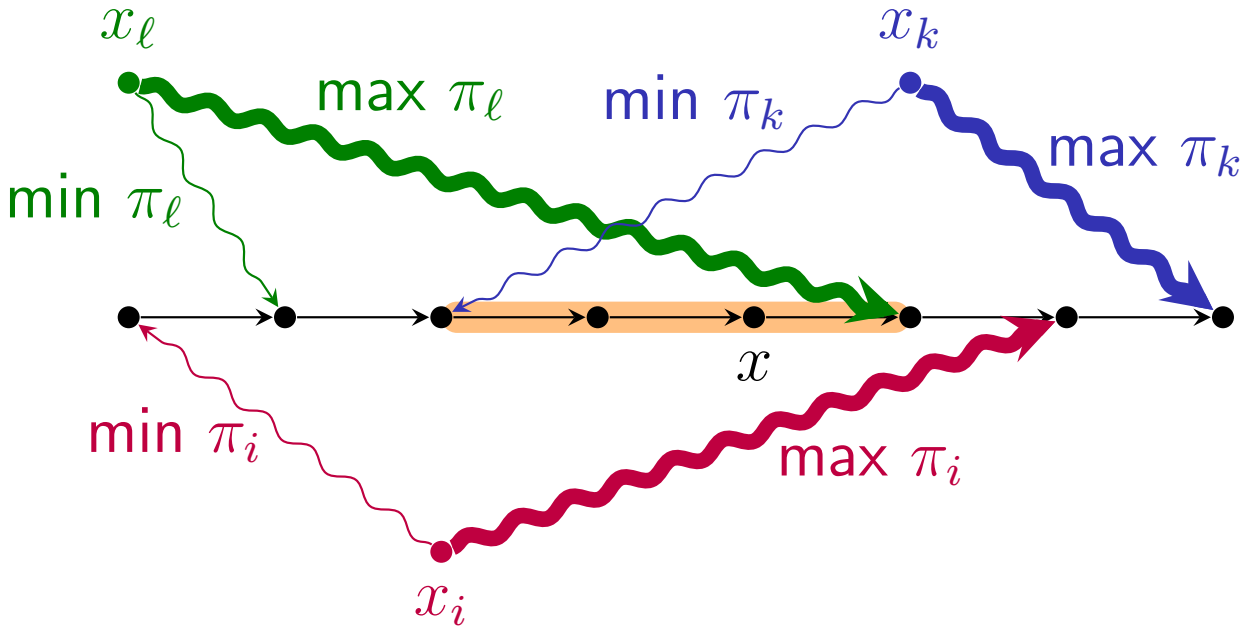

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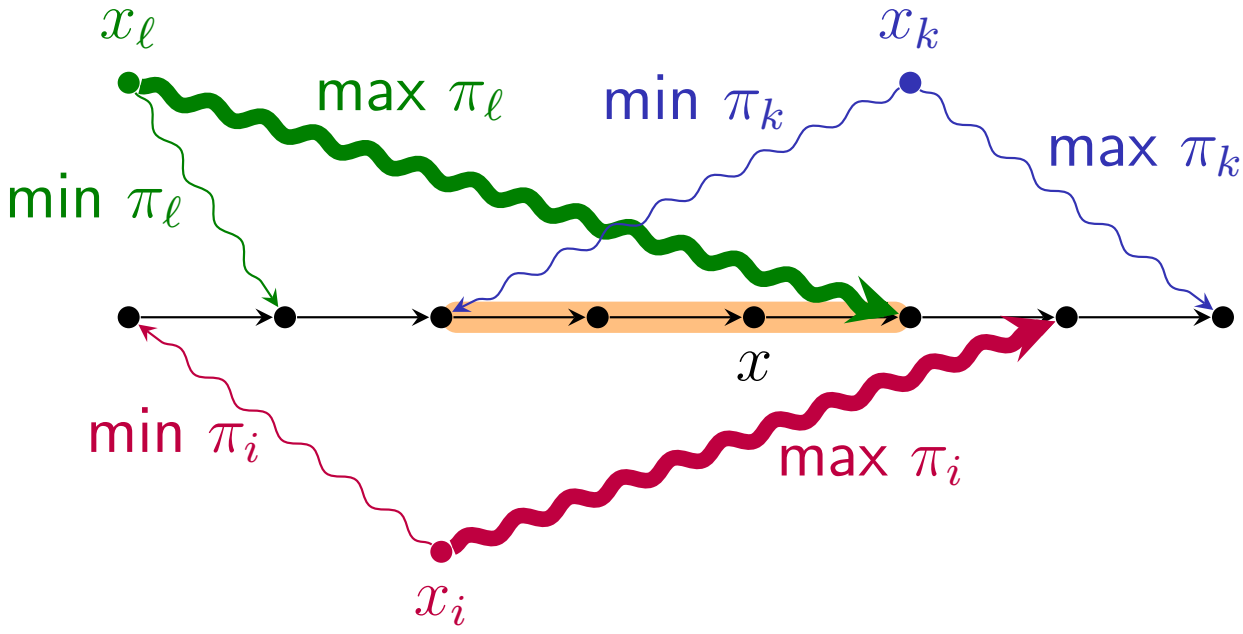

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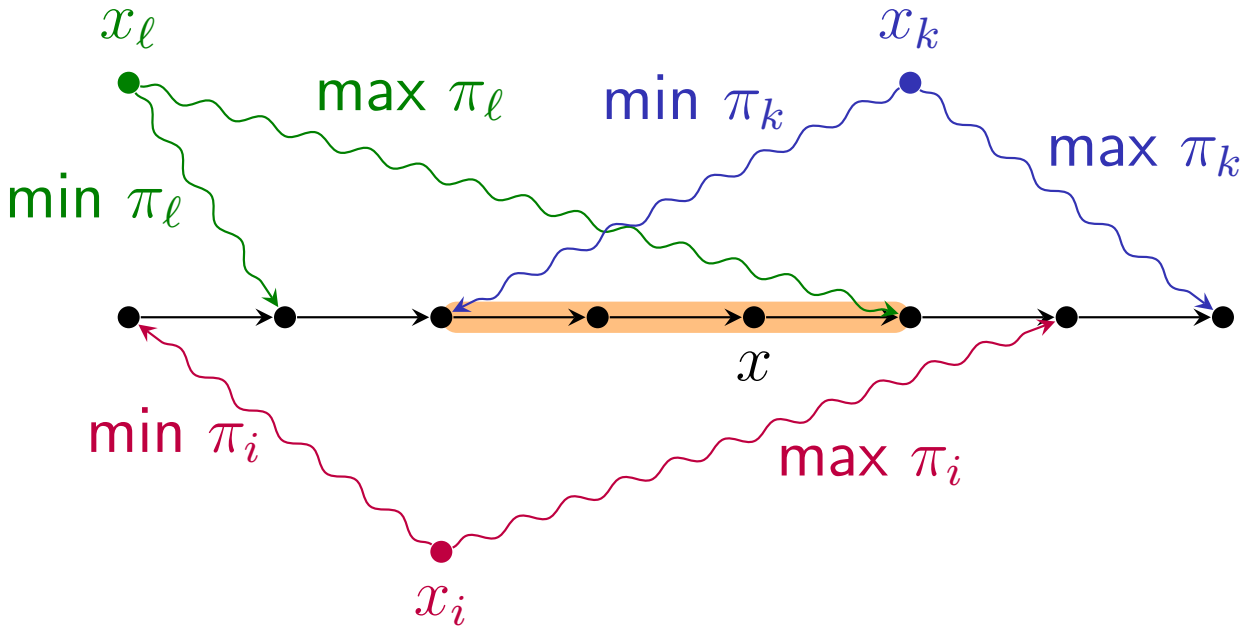

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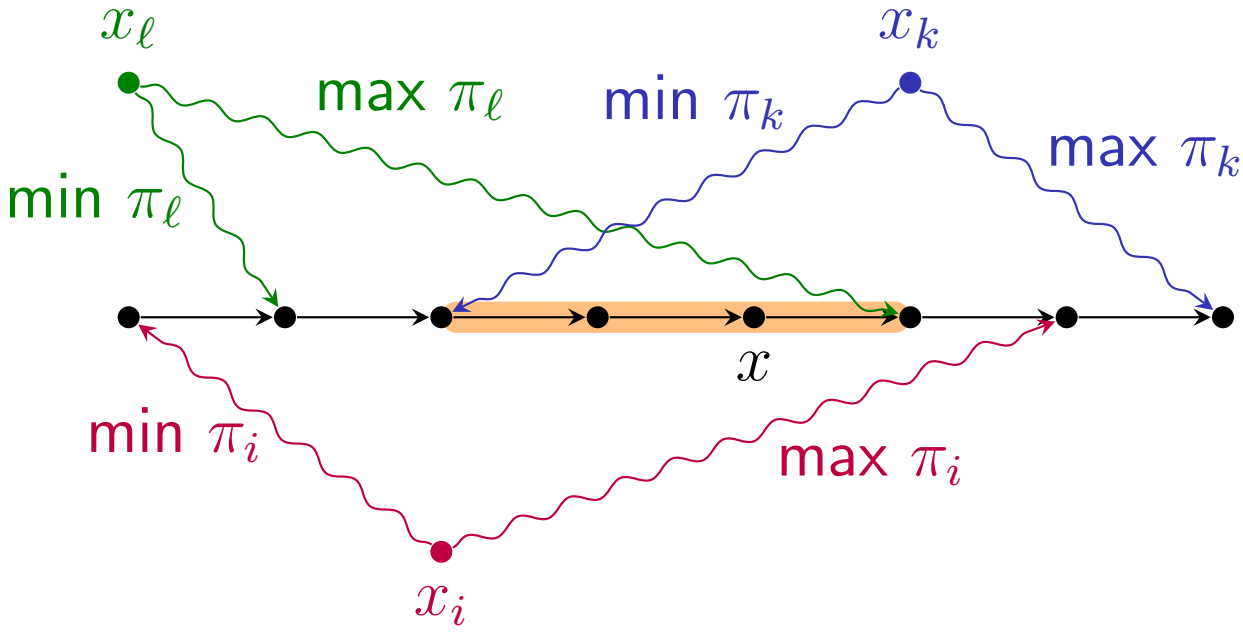

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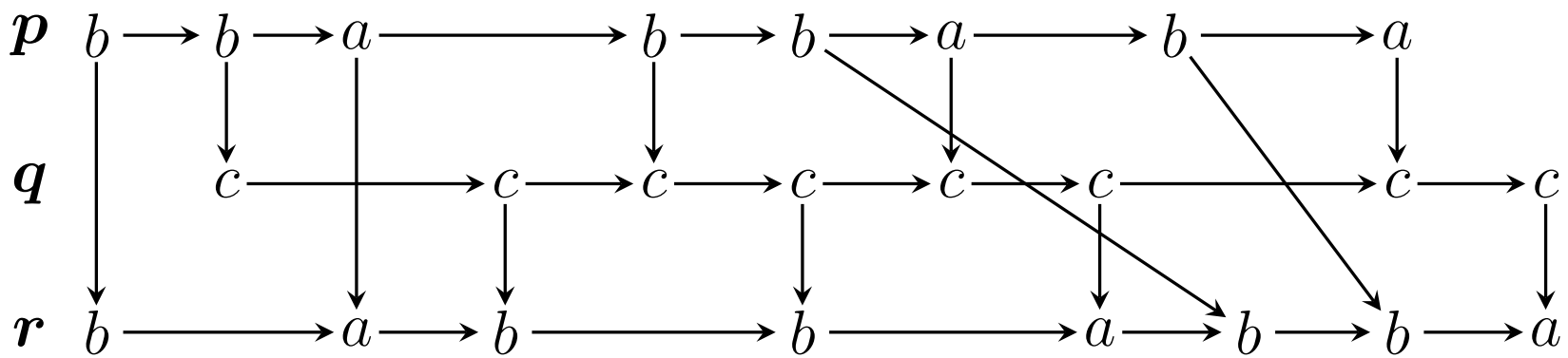

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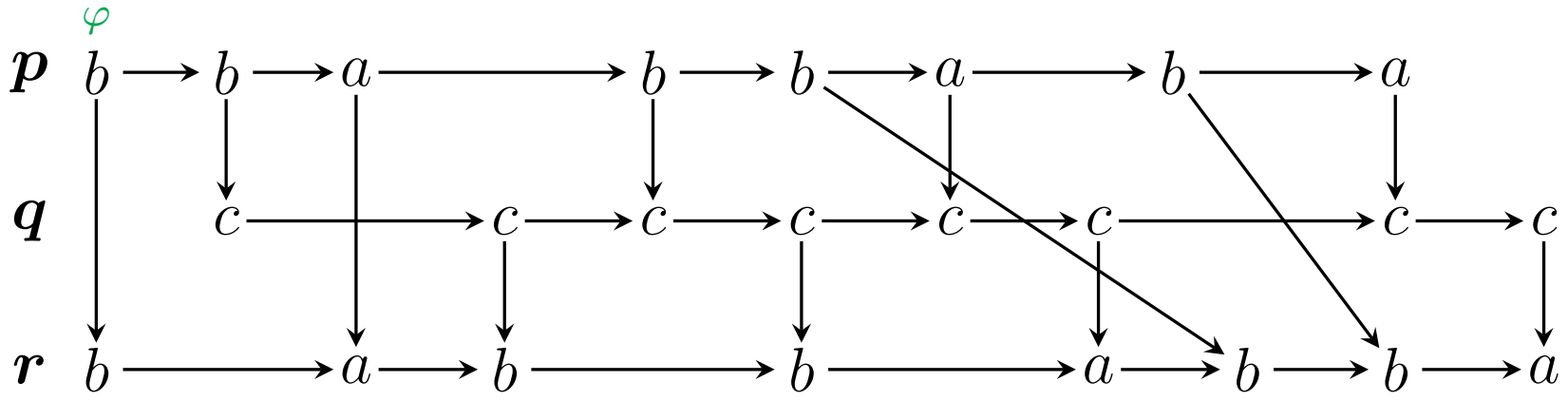

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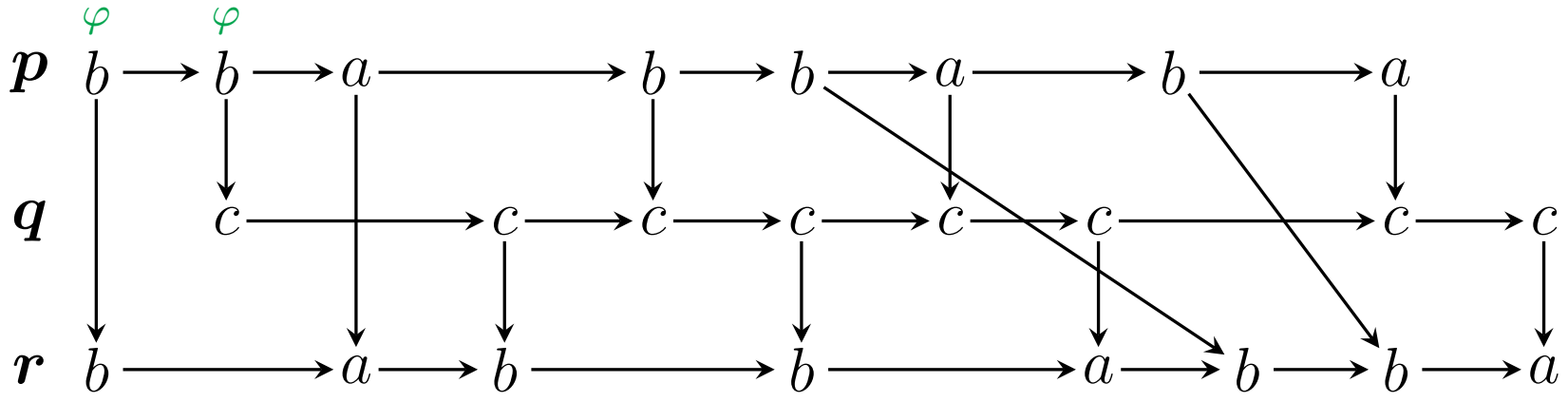

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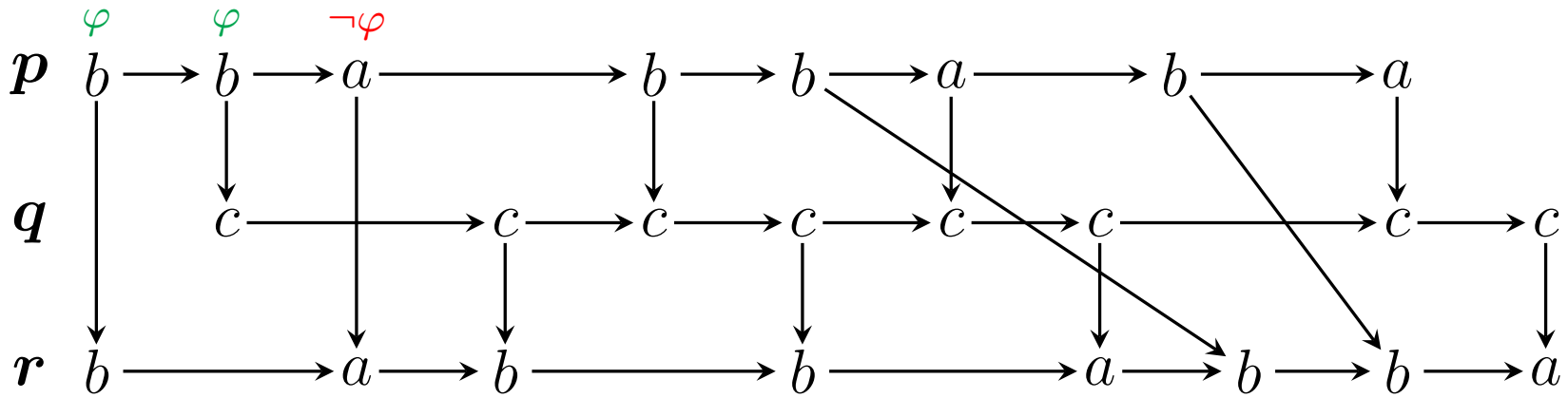

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Any event formula $\varphi \in PDL_{sf}$ can be translated into a CFM which determines for each event whether φ holds.

Proof: By induction.

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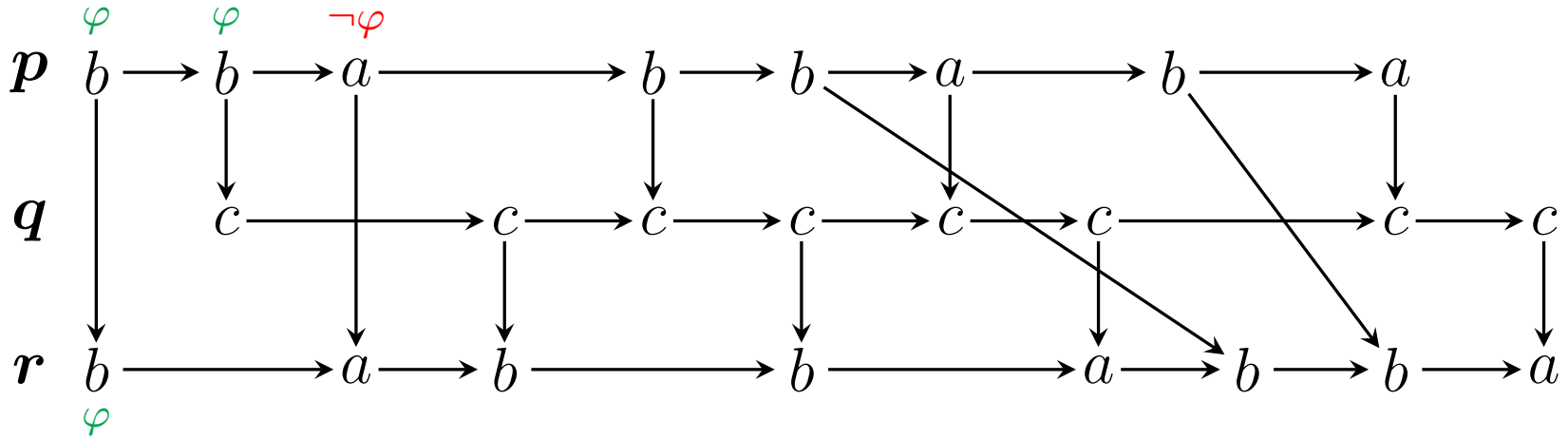

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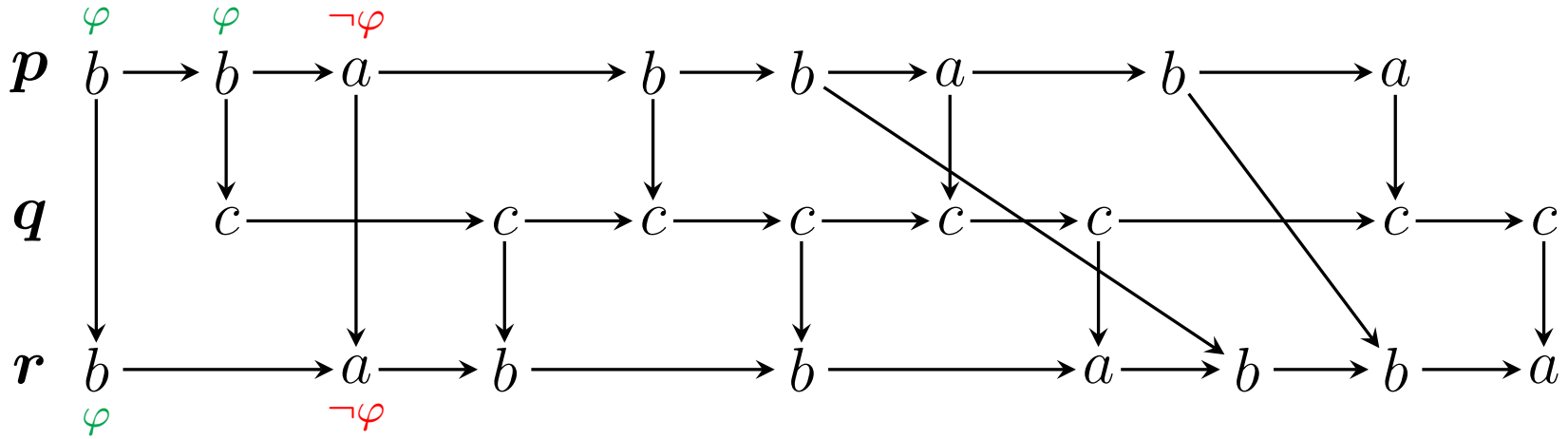

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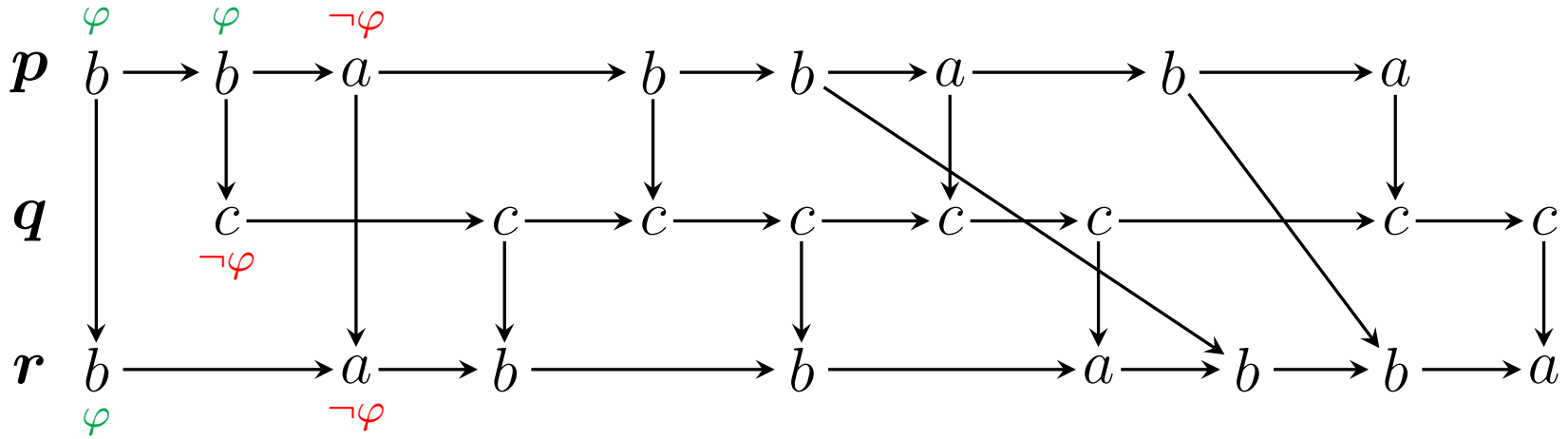

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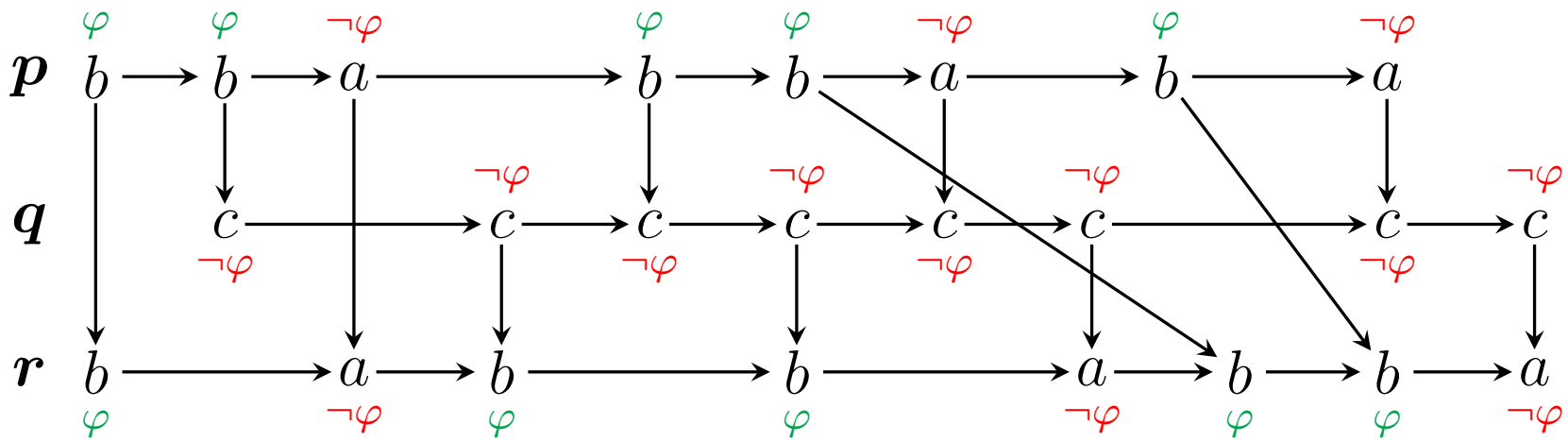

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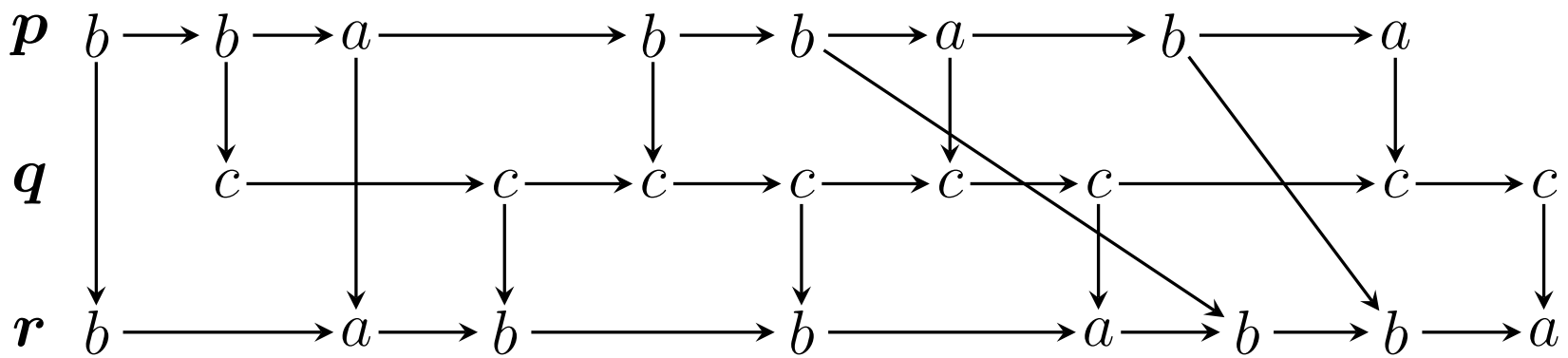

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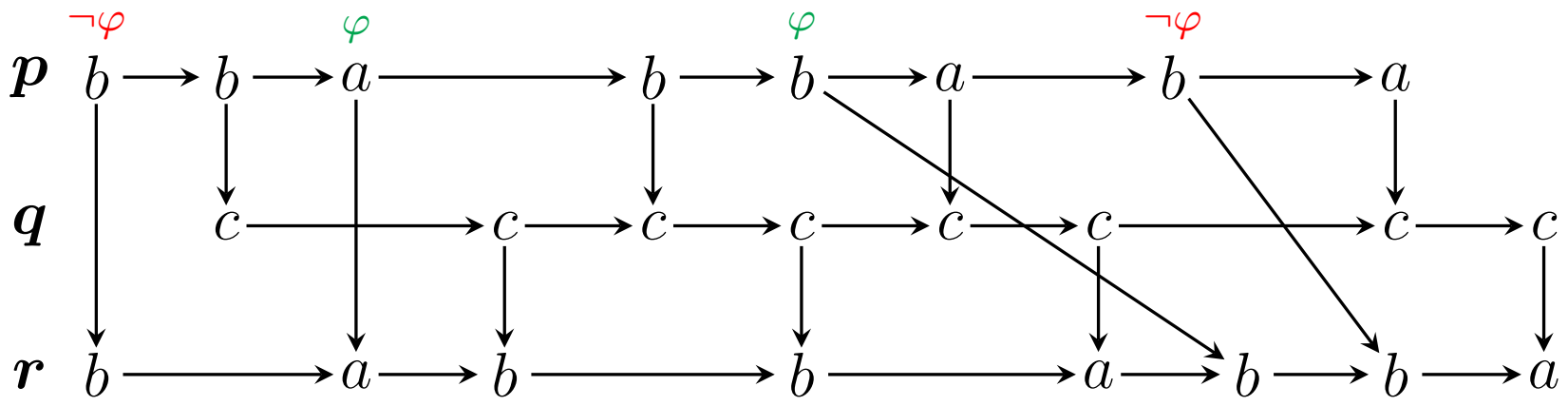

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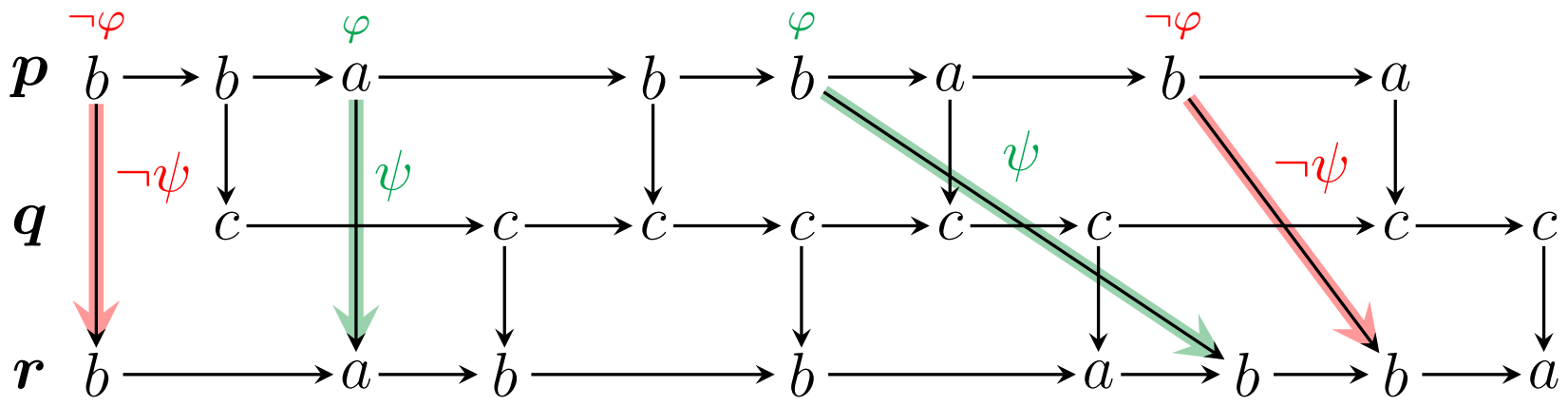

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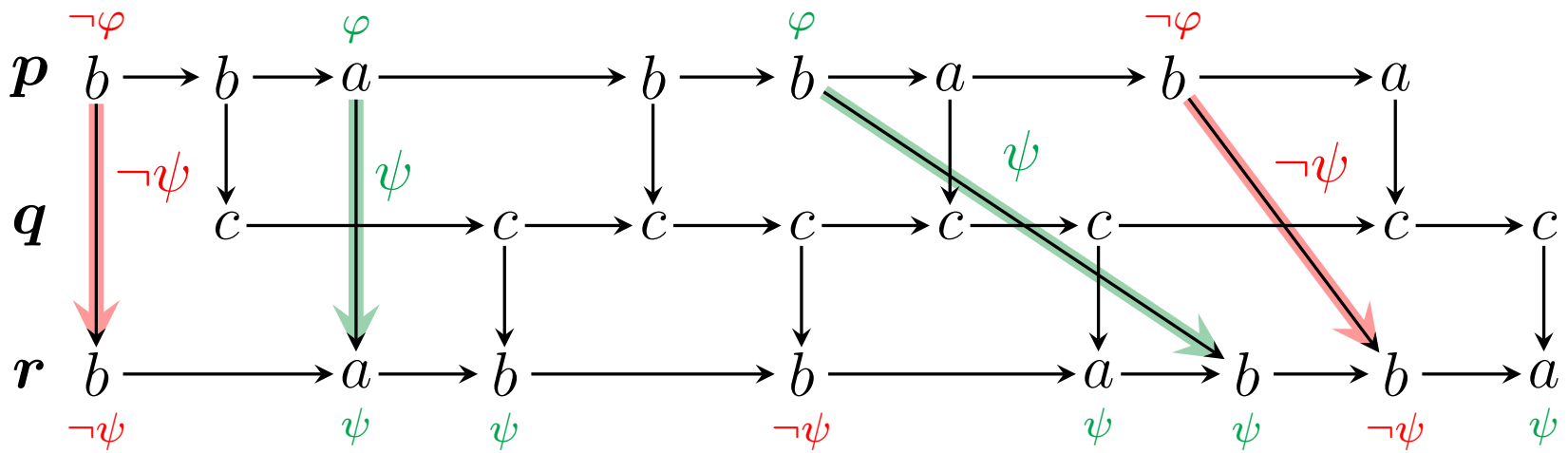

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- ...
- Only difficult case: $\varphi = \mathsf{Loop}(\pi)$

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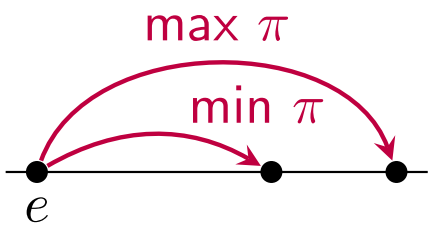

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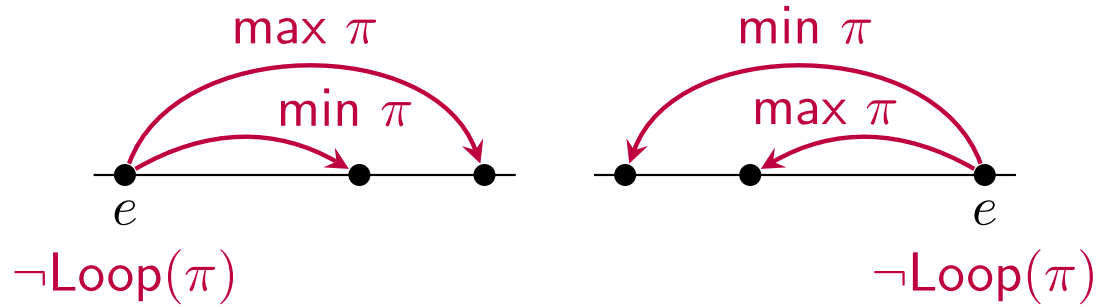

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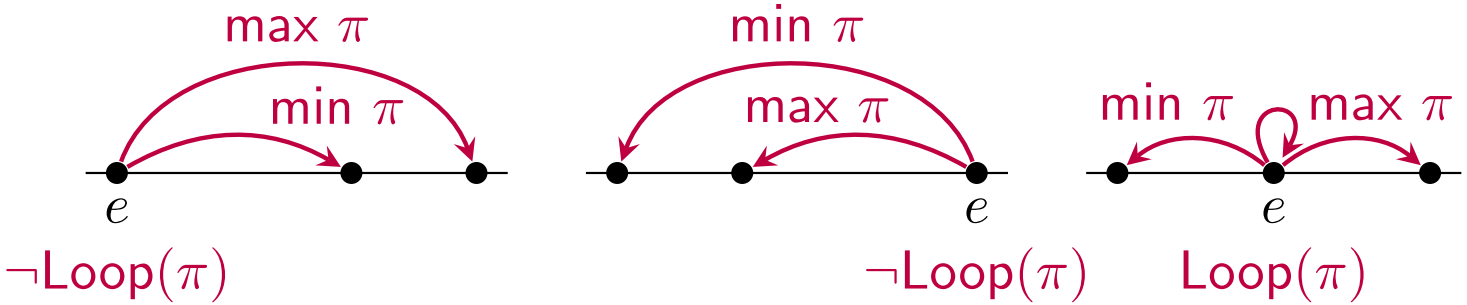

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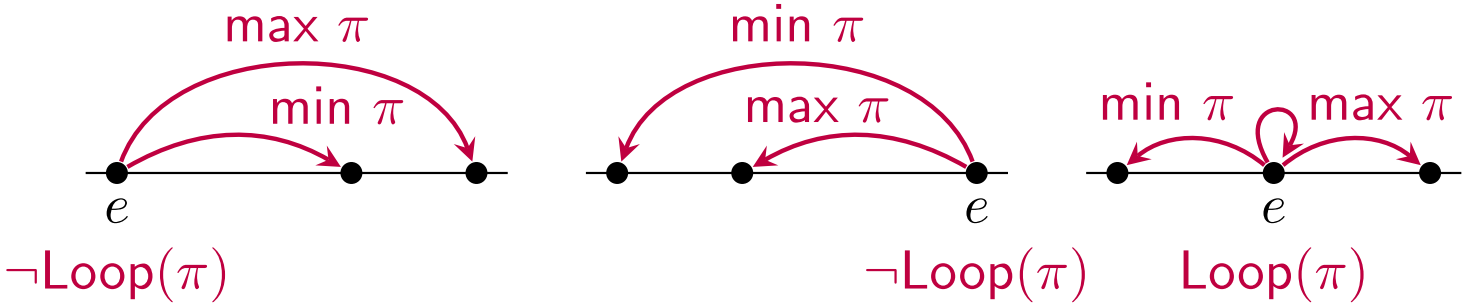

- To know which of these cases applies, it is enough to evaluate formulas $\mathrm{Loop}(\min\ \pi')$ and $\mathrm{Loop}(\max\ \pi')$.

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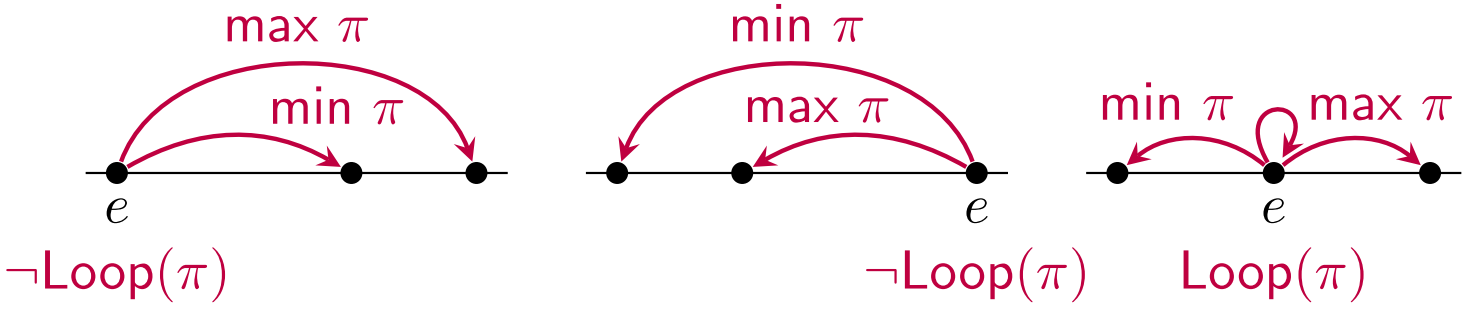

- To know which of these cases applies, it is enough to evaluate formulas $\mathrm{Loop}(\min \pi')$ and $\mathrm{Loop}(\max \pi')$.

1) Translate $\mathrm{Loop}(\min \pi')$ to $\mathrm{Loop}(\max \pi')$ into CFMs.
2) Use this to evaluate $\mathrm{Loop}(\pi)$ from left to right.

CFM for $\varphi = \mathsf{Loop}(\max\ \pi)$

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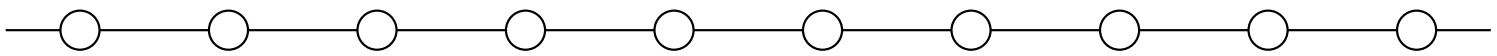
- Guess for each event whether φ holds.



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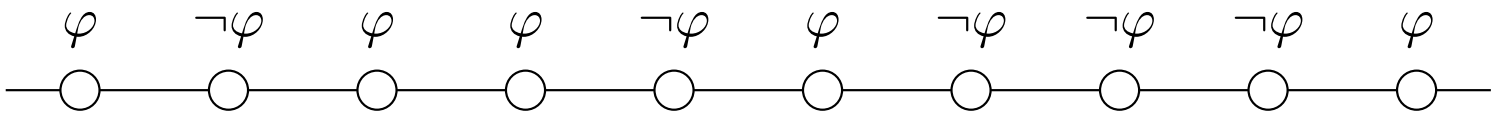

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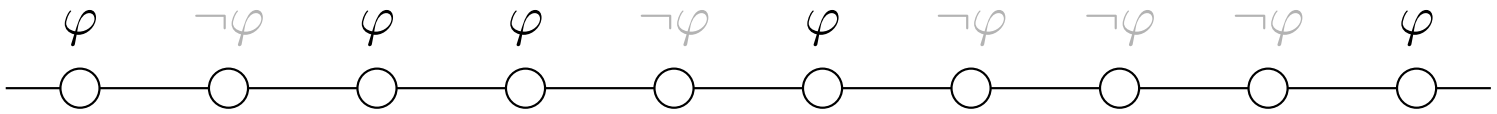

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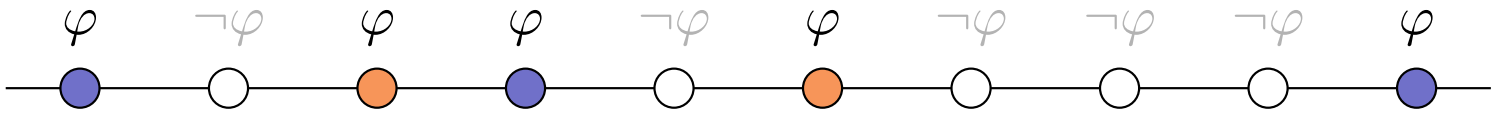

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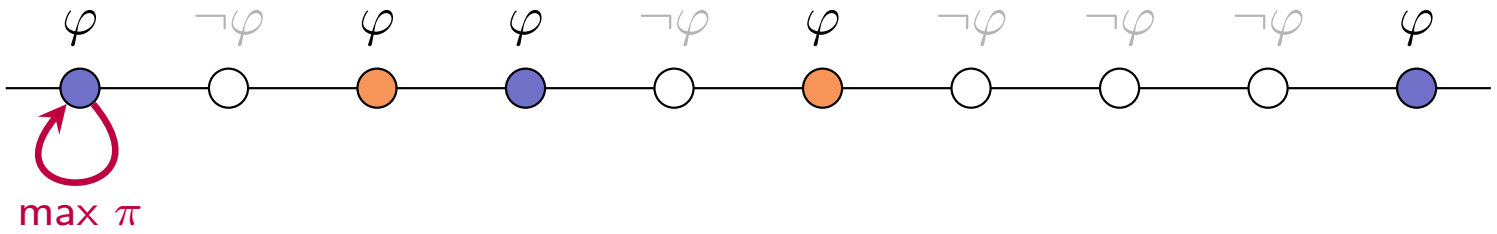

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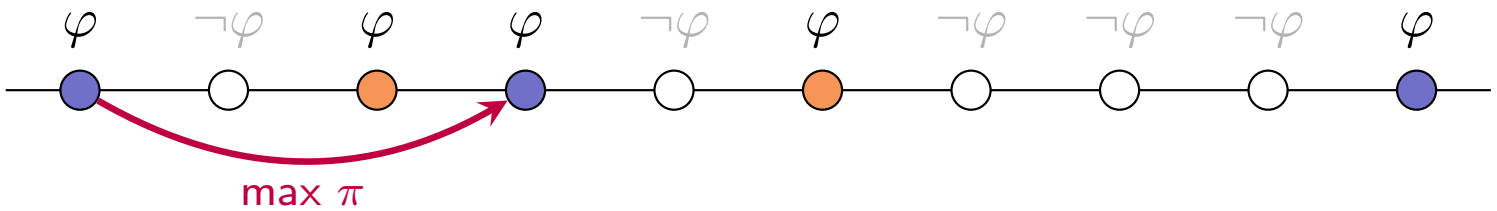

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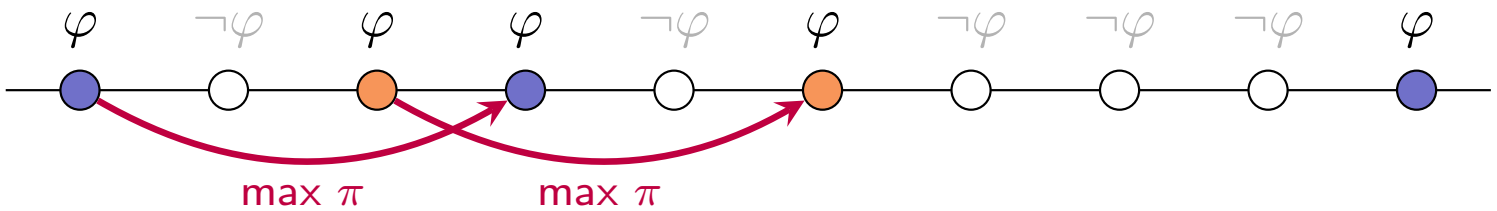

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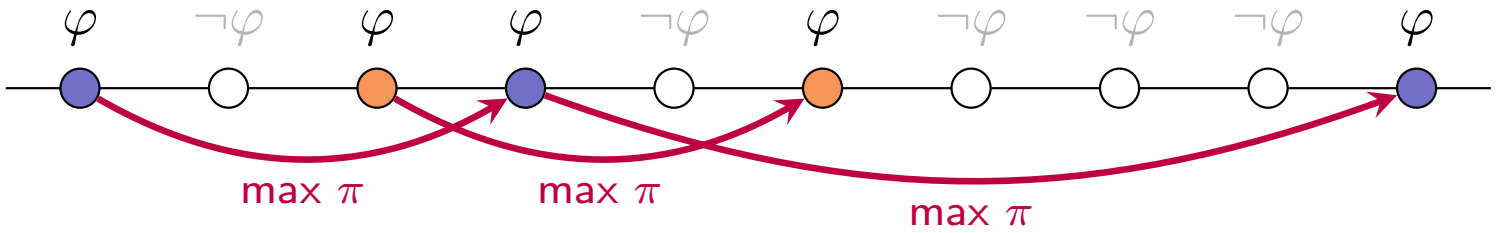

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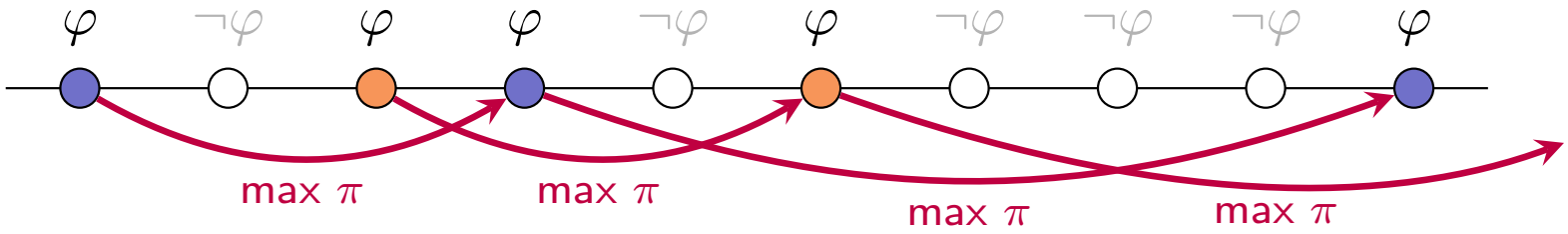

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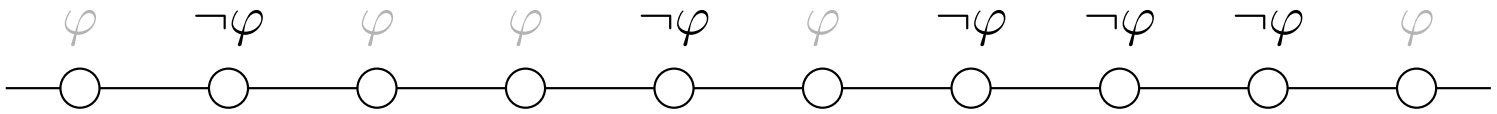

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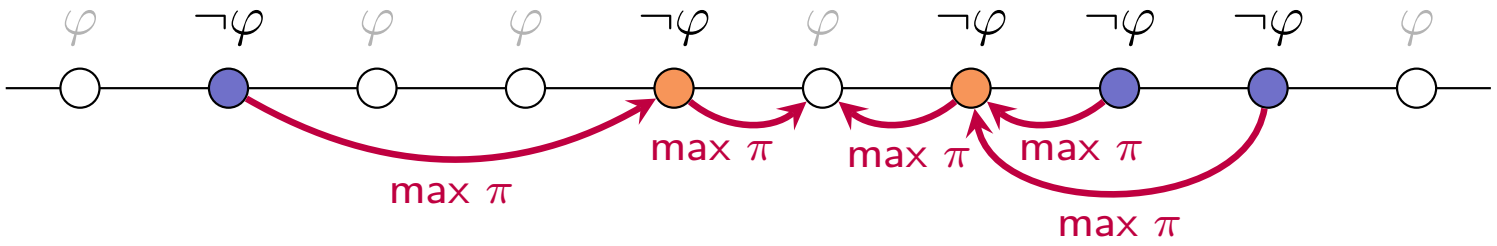

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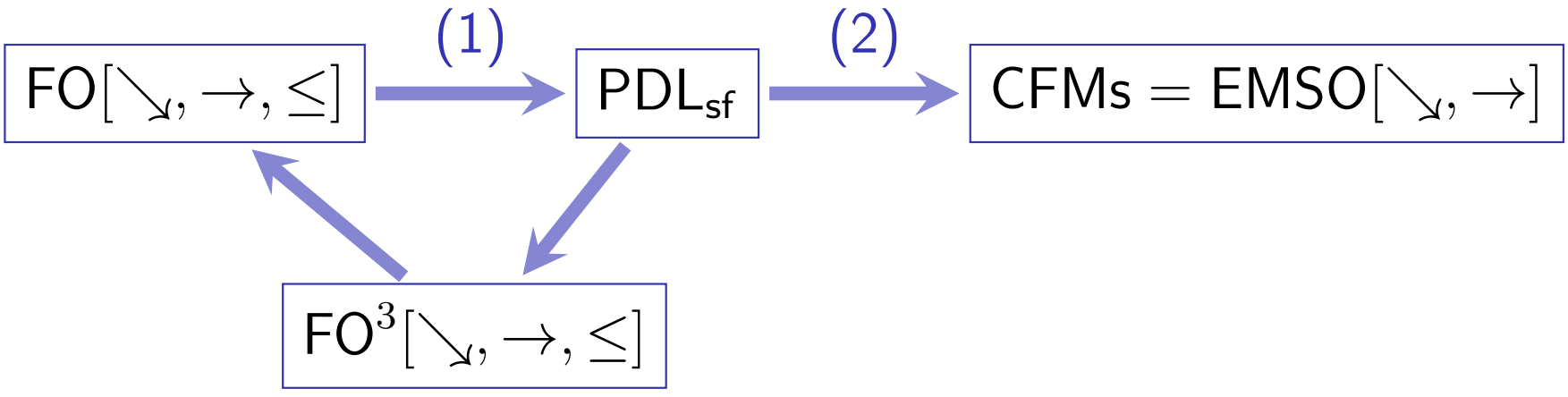
Summary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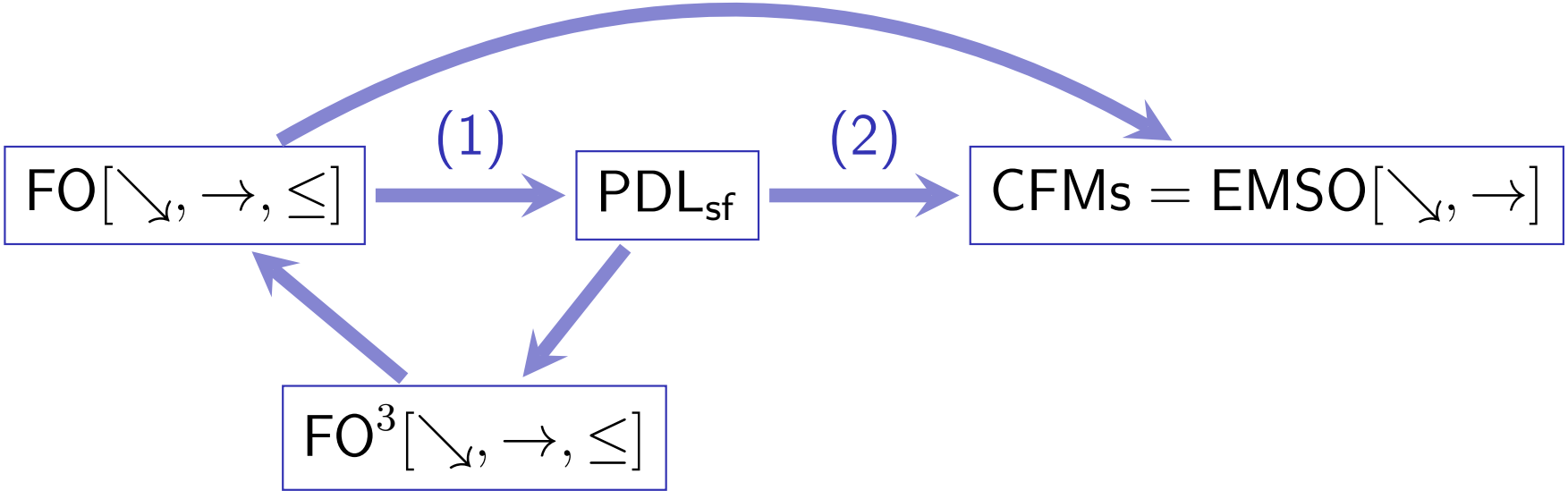
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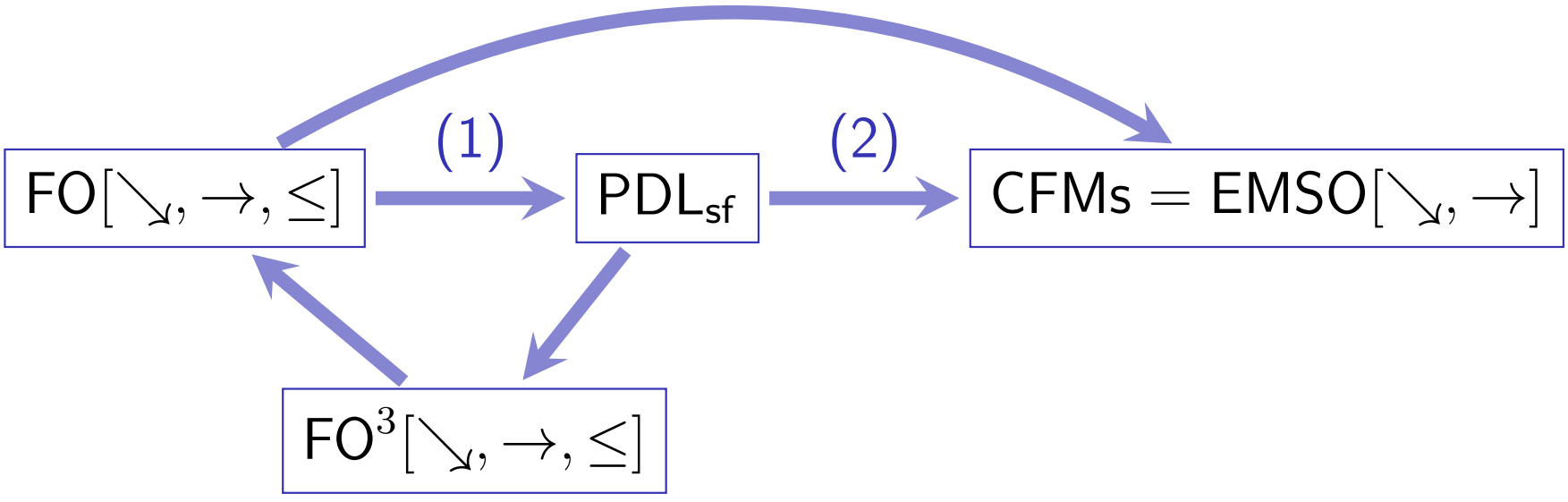

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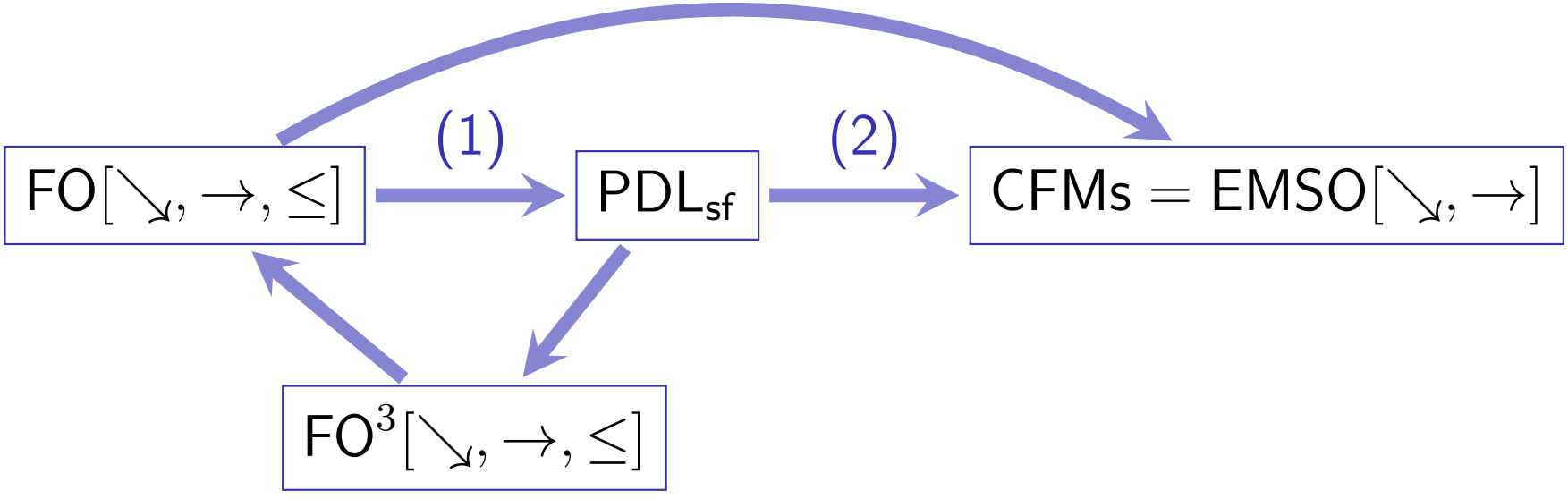

- FO sentence is a positive boolean combination of formulas $\exists x.\Phi(x)$ or $\neg\exists x.\Phi(x)$.
- Both types of formulas can be evaluated using the CFMs for $\Phi(x)$.

Conclusion

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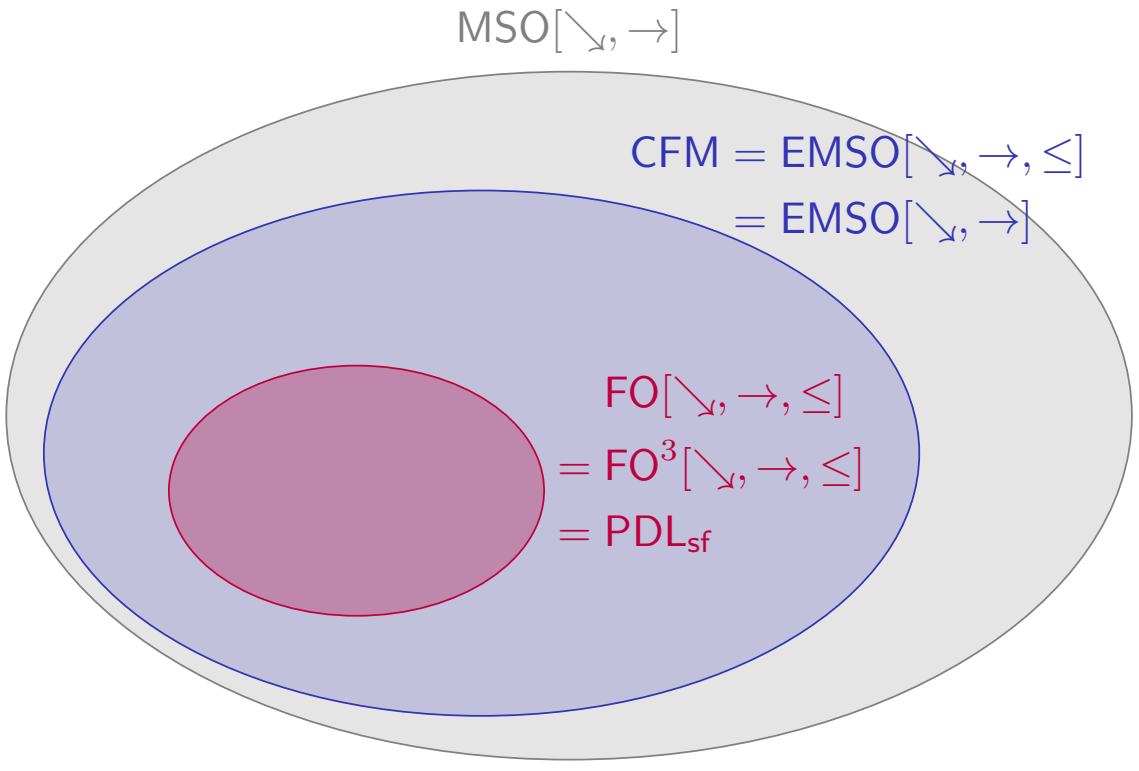

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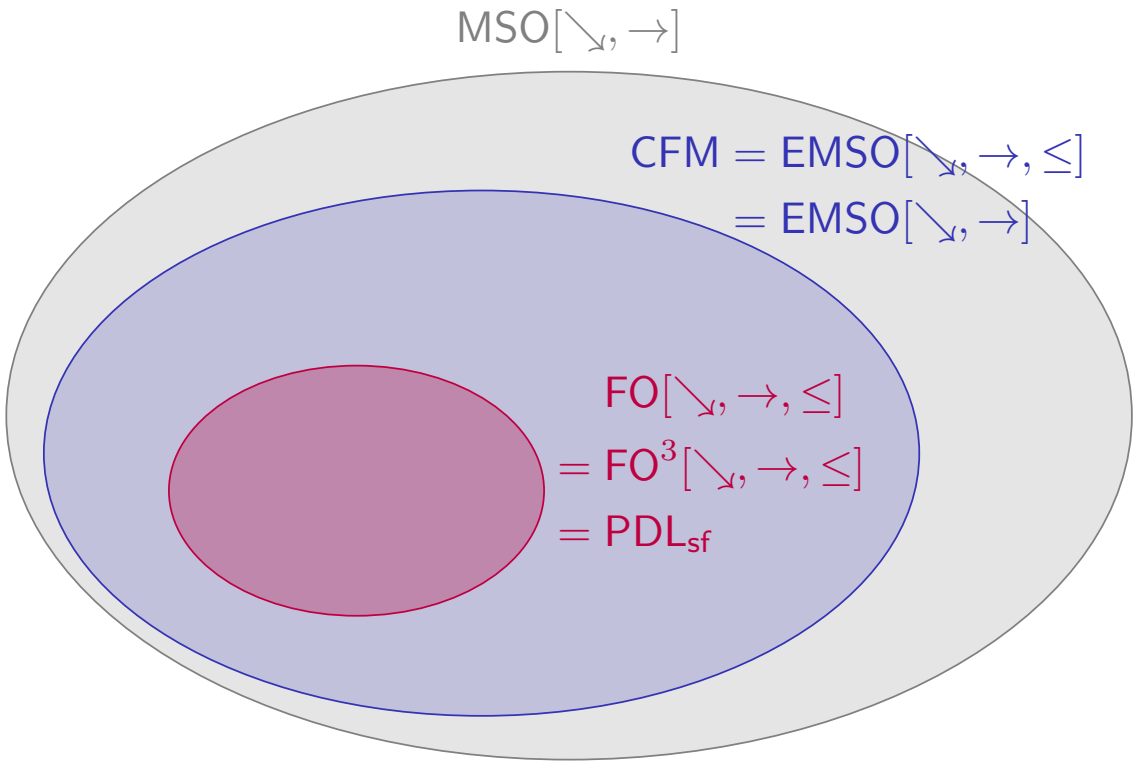

Open questions:

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- PDL (with star but without Loop) vs. CFM?

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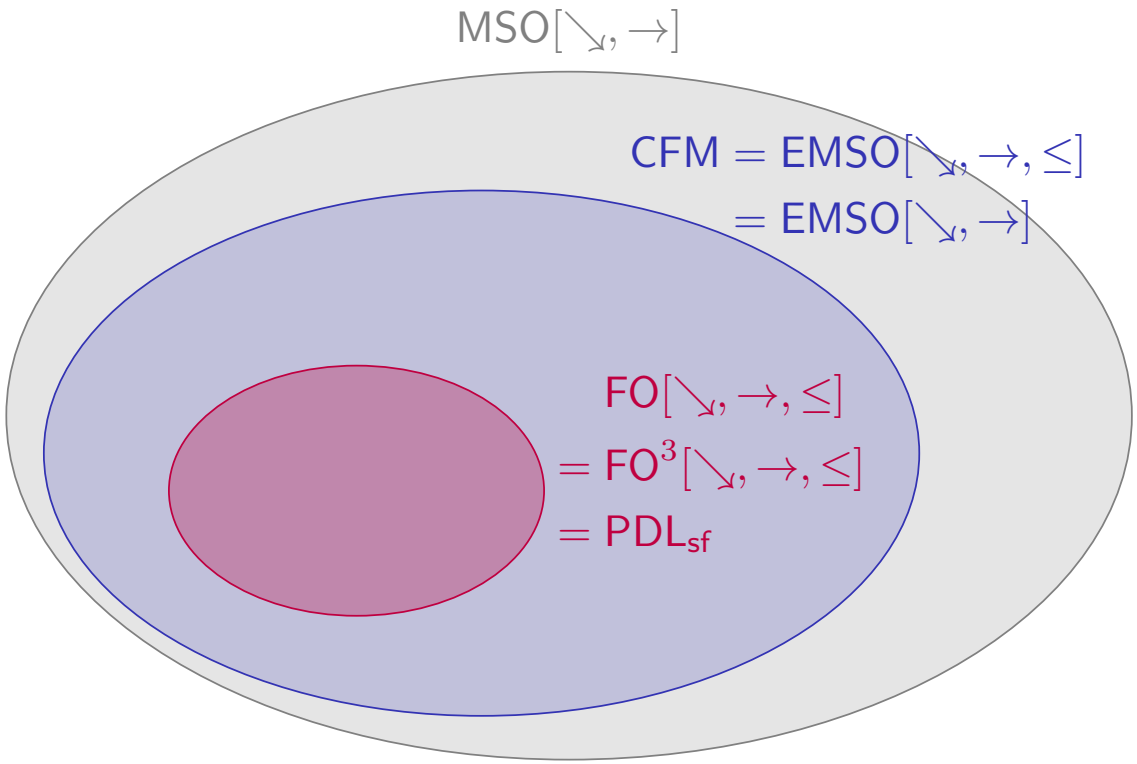

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Thank you!